

Vitruvius on Architecture:  
A Modern Application and Stability Analysis of Classical  
Structures

by

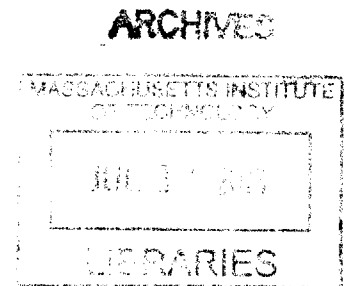
Ana S. Escalante

Submitted to the Department of Mechanical Engineering in Partial  
Fulfillment of the Requirements for the Degree of  
Bachelor of Science in Mechanical Engineering

at the

Massachusetts Institute of Technology

June 2013



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**ABSTRACT**

Imperial Rome has left numerous legacies, the most well-known being its literature and monuments. Though many monuments, such as the Pantheon, are well-preserved, in cases where little physical evidence remains, historians can often use literary sources to inform reconstruction efforts. For more technical studies of Roman construction, technical literature is rare and the contemporary awareness of such literature even less known. When Vitruvius wrote *De architectura*, he did not intend for it to be a manual for instruction but rather a central source of general architectural knowledge. Directly aimed at architects, contractors, and other individuals involved in the design and construction of buildings, *De architectura* provides insight into contemporary technical knowledge. One aim of this thesis is to identify the presence of Vitruvian knowledge in imperial Roman structures. *De architectura* was written during the time of Augustus, therefore Augustan monuments show the immediate impact and relevance of the knowledge presented by Vitruvius. Almost a century later, architectural innovation was a hallmark of Hadrian's reign, but a study of Hadrianic structures demonstrates the longevity of *De architectura*. A structural analysis of the Teatro Marittimo and Sala dei Filosofi in Hadrian's villa at Tivoli, both influenced by Vitruvian precepts, was carried out to characterize the load distribution in supporting structures. The results of this analysis demonstrate that although Vitruvius gave no quantitative support for his guidelines, his suggestions are structurally sound, even by modern engineering standards.

Thesis Supervisor: William Broadhead  
Title: Professor of History





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### 1.1 Imperial Rome: Architecture & Construction

The Roman Empire has left many legacies, the most tangible of which are the buildings which hint at the skill and innovation of Roman architects. Most of the buildings which still stand today were constructed for use as public spaces and their construction contracts were bid upon by independent contractors. When Augustus became emperor, he developed an improved administrative structure to deal with the construction and maintenance of public buildings and structures. The new infrastructure behind such activities made construction projects more efficient and the labor more organized.<sup>1</sup> Construction projects operating under greater accountability and efficiency enabled the innovation that distinguished Roman architecture.

The separation of architecture from engineering is a modern development. In the Roman republic and empire, architecture and engineering were not separate entities. In designing buildings, ancient architects had to be competent in architectural design and had to possess technical skills relating to the stability and safety of a building. The creativity of a design was limited by structural concerns as well as the standards of appearance for public buildings. In contrast to modern times, architects also led the construction of buildings which required them to be knowledgeable about materials selection based on availability, workability, and strength constraints.<sup>2</sup>

Historians have had to use surviving structures to figure out the extent of the builders' knowledge of construction and architecture. Evidence from literary sources provides the context necessary to show the evolution of this technical knowledge. One such source, Vitruvius' *De architectura*, provides the most complete description of an ancient architect's competencies though it is biased by Vitruvius' opinion of what an architect should know.<sup>3</sup> He chose to focus on the basic principles of architecture and provides anecdotal evidence of their significance. However, from a general survey of architecture during Vitruvius' lifetime, it is plain to see that not everyone attached such great significance to each architectural element. Nevertheless, since *De architectura* was the most comprehensive text on architecture, it continued to be copied and studied, even into the medieval ages.<sup>4</sup>

Imperial Roman buildings, though rarely completely preserved, offer a frontier for modern engineers. The challenges of reconstructing the remains and examining the evidence under preservationist constraints forces modern engineers to adjust their method. Furthermore, engineers must reverse engineer their way from the physical evidence to the construction practices of ancient Rome. In this quest to understand ancient Roman engineers, Vitruvius' *De architectura* provides both the breadcrumbs and perspective to help guide modern engineers in their analysis of ancient knowledge. The objective of this thesis is to evaluate the application and merit of the architectural precepts set forth in the work *De architectura*. A brief explanation of the author will explain the context and relevance of *De architectura* to Roman construction.

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<sup>1</sup> Lancaster, *Concrete Vaulted Construction in Imperial Rome*, 18-19.

<sup>2</sup> Lancaster, *Concrete Vaulted Construction in Imperial Rome*, 19-20.

<sup>3</sup> Vitruvius, *De architectura*, 1.2.

<sup>4</sup> McEwen, *Vitruvius*, 156-157.

## 1.2 Vitruvius

Thought to have lived from about 80 BC to about 15 BC, Vitruvius was a man of many talents.<sup>5</sup> Vitruvius' life experiences began under Julius Caesar and continue into the reign of Augustus. He spent his early life in military service under Julius Caesar.<sup>6</sup> By his own words, it was in this capacity that Vitruvius developed skills concerning the design and construction of contraptions used for war. After his military service, Vitruvius turned his engineering skills towards a career in architecture. Thus far, only one building has been confidently accredited to Vitruvius- the basilica he claims to have built under Augustus.<sup>7</sup>

Vitruvius, however, is best recognized as an author. His best known and only surviving works are the ten books that comprise *De architectura*. Though Vitruvius professes that he is merely compiling previous knowledge, the work reveals the extent of Vitruvius' own knowledge and understanding of architecture and engineering. Vitruvius often explains that *De architectura* was written as a service for the benefit of Augustus, other architects, and all the Roman people.<sup>8</sup> He believes that gathering and simplifying architectural knowledge will help Augustus to better understand architecture and build monuments worthy of recognition in posterity.<sup>9</sup> He clearly intended, or at least strongly hoped, that *De architectura* would be used widely to standardize and improve Roman architecture.

In Book I of *De architectura*, Vitruvius describes how an architect, educated in technical areas such as geometry, should be able to communicate well- in speech and composition, so that the final product will reflect their competency.<sup>10</sup> He elaborates on the subsets of architectural education in subsequent books. Whether or not Vitruvius' description was accurate at the time it was written, as awareness of *De architectura* spread, architecture began to reflect more of his precepts.

Classical Roman architecture grew from a foundation of Greek architectural tenets. These roots are most obvious when studying the foundations of Roman buildings and the design of columns.<sup>11</sup> In explaining the derivation of proportions in Book III, Vitruvius carefully notes that mathematics is rooted in Greek philosophy and geometry. He also credits Greek architects in Books IV and V where Vitruvius delves into more proportions for columns and temples.<sup>12</sup> This reinforces the belief that Vitruvius is not creating new architectural principles so much as he is explaining and shaping old architectural knowledge.

In the preface to Book I, Vitruvius dedicates *De architectura* to Augustus for the benefit of his education and construction projects.<sup>13</sup> Augustus (63 BC to 14 AD), first emperor of the Roman Empire, is memorable for bringing peace to the Roman Empire and for numerous other

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<sup>5</sup> McEwen, *Vitruvius*, 1-13.

<sup>6</sup> Vitruvius, *De architectura*, 1.preface.2.

<sup>7</sup> Vitruvius, *De architectura*, 1.preface.2.

<sup>8</sup> McEwen, *Vitruvius*, 130-154.

<sup>9</sup> Vitruvius, *De architectura*, 1.preface.2.

<sup>10</sup> Vitruvius, *De architectura*, 1.1.

<sup>11</sup> Malacrino, *Constructing the Ancient World*, translated by Hyams, 111-112.

<sup>12</sup> McEwen, *Vitruvius*, 54-70.

<sup>13</sup> Vitruvius, *De architectura*, 1.preface.2.



accomplishments which Augustus lists in the *Res gestae divi Augusti*.<sup>14</sup> He radically changed the Roman landscape through countless building projects. According to Suetonius, the Roman historian, Augustus even boasted that he changed Rome from a city of brick to a city of marble.<sup>15</sup> Though Cassius thought that Augustus was alluding to the renewed strength of the Roman Empire (and he could be right), Augustus may have been referring to the renewed strength of Roman buildings, many of which actually used marble.<sup>16</sup> Whatever the case, Augustus' reign was marked by peace which facilitated a revival of Roman architecture and construction. He also created a trend which most future emperors continued to follow. This trend was built on the idea that building projects were benefits to the Roman citizens through which an emperor displayed and shared the strength and wealth of the Roman Empire. This was just one channel through which an emperor could care for his people and earn the title which Augustus was most proud of: *Pater Patriae*.

Almost a century later, following Emperor Trajan's lifetime of territorial expansion and the consequent acquisition of even more imperial wealth, Hadrian's peaceful reign began another revival of Roman architecture and construction. Hadrian ruled from 117 AD to 138 AD and is well known for his extensive travel and patronage of the arts.<sup>17</sup> Hadrian sponsored many building projects but two stand out in grandeur. The famous dome of the Pantheon, in its surviving configuration, was rebuilt by Hadrian in 126 AD after the original structure, built by Marcus Agrippa, was destroyed in the Great Fire of 80 AD. The Pantheon, literally 'all gods,' remains the world's largest unreinforced concrete dome.<sup>18</sup> The largest project undertaken by Hadrian was the construction of the villa at Tivoli. Built as a personal retreat within a retreat, the villa at Tivoli contains elements of the previous traditional Roman architecture as well as innovations which marked later Roman architecture. The villa is also significant because rather than acting solely as sponsor of the project, Hadrian appears to have flexed his own architectural skills and broken away from the standard designs of Vitruvius.<sup>19</sup>

The analysis in this thesis used this historical background to choose structures that provided opportunities to evaluate the relevance of Vitruvius at the time when *De architectura* was written, as well as a much later time distinguished by architectural innovation.

### 1.3 Thesis Overview

This thesis is an analysis of *De architectura* with an eye to context and application of the proposed architectural precepts and will progress in three parts: a critical analysis of *De architectura*, the development of an engineering model of structural stability, and an in-depth analysis of specific structures. The evaluation of *De architectura* has two fronts but both require an analysis of the work itself. *De architectura* is a comprehensive work that encompasses an astonishing variety of engineering topics in addition to architecture. The analysis of *De architectura* begins by identifying Vitruvius' intentions and motivations to contextualize the

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<sup>14</sup> McEwen, *Vitruvius*, 8.

<sup>15</sup> Suetonius, *Augustus*, 28.

<sup>16</sup> Cassius, *Rome*, 56.30.3.

<sup>17</sup> Ricotti, *Villa Adriana*, 13.

<sup>18</sup> Lancaster, *Concrete Vaulted Construction in Imperial Rome*, 192.

<sup>19</sup> Ricotti, *Villa Adriana*, 24.

outlined rules and eventual impact. Then, the content of Books I-V is studied to identify the precepts which claim to have engineering reasoning.

In Chapter 2, *De architectura* is summarized and the focus narrowed to the sections most relevant to structural stability.

Chapter 3 outlines the development of engineering models that were used to determine the validity and merit of the rules by which Vitruvius bounds architecture. This chapter will provide a brief summary of the materials available to imperial Roman architects. A short discussion of the mechanics of masonry structures will begin the derivation of force equilibrium equations for chosen structural elements.

Chapters 4 and 5 contain the applied analysis of Vitruvian architecture guidelines with respect to actual structures. On one hand, the objective is to determine the relevance of *De architectura* to Roman architecture. Therefore, an analysis is first applied to Augustan buildings to determine the immediate relevance of *De architectura* to architectural projects. The Temple of Divus Julius is the subject of this case study. Analysis of Hadrian's Villa and the Pantheon provides evidence for the longevity of *De architectura*. The second objective is to determine the engineering merit of *De architectura*. Using data from surviving, Hadrian-era buildings with confirmed Vitruvian influences, the results from the analytical models developed in Chapter 3 will be presented in Chapter 5. In this section, the models will be applied to the Pantheon as confirmation of the models' accuracy. Then the models will be applied to the Teatro Marittimo and Sala dei Filosofi in Hadrian's villa.

Chapter 6 ties together the results of both types of analysis to discuss the relevance and robustness of *De architectura*.

## 1.4 Previous Scholarship

Classical and imperial Roman architecture has been the focus of numerous studies. Most studies apply modern analytical tools to ancient structures to understand the structural mechanics of surviving buildings, to learn about the construction techniques used by the builders, or to trace architectural evolution. Studies of structures that no longer survive often use the literary record to accomplish the same goals. In contrast, this thesis uses modern analytical tools to evaluate an ancient literary work with respect to the application in imperial Roman structures. Nonetheless, without the extensive work of the following scholars, the scope of such an evaluation would far exceed an undergraduate thesis.

Pierre Gros is the foremost scholar on Vitruvius. He uses both literary sources and surviving structures to characterize the predominant architectural style of late Republican and early Imperial Rome. He also has compared the architecture of buildings with the rules set forth by Vitruvius as part of the characterization of Augustan architecture.<sup>20</sup>

Eugenia Salza Prina Ricotti has done similar work as Gros but with a focus on Hadrian-era structures. She has been able to use the characterization of that era of architecture to identify

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<sup>20</sup> Gros, *Le Projet de Vitruve* [1993] and *L'Architecture Romaine* [2001].

the work of individual architects. She has also closely studied the bricks in Hadrianic buildings to determine the strength of different materials, construction methods, and configurations.<sup>21</sup>

Lynne Lancaster pioneered thrust line analysis of concrete vaulted structures. Her methodology lines allowed for simpler and more accurate characterization of dome stresses by defining thrust lines. This also allowed her to predict deformation and failure of vaulted structures by finding hinge locations with minimum and maximum thrust lines. Her analysis of the Pantheon's dome formed the basis for the model used in this thesis.<sup>22</sup>

Jacques Heyman manages to condense an astonishing amount of masonry research into simple models of masonry structure mechanics. He succinctly qualifies the conservatism of the assumptions made about the material properties of rock and explains the limits of boundary conditions in the mechanics of rock structures.<sup>23</sup>

Relatively few historical structures have been measured and catalogued in fine detail. Scholars often have to sacrifice resolution in favor of scope or vice versa. Mathias Uebliker favored resolution over scope. His work on the Teatro Marittimo in Hadrian's villa at Tivoli contains detailed measurements of most of the remains. These measurements allowed him to analyze the architecture in fine detail. He also characterized Vitruvius' guidelines for column capitals numerically for comparison with the measurements of the columns in the Teatro Marittimo.<sup>24</sup>

In contrast, the architects Robert Mangurian and Mary-Ann Ray undertook the substantial project of measuring and cataloguing the entirety of Hadrian's villa. Over the course of several years, they measured every feature of the structures in Hadrian's villa and have graciously provided this data to other scholars wishing to conduct analyses of the same structures.<sup>25</sup>

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<sup>21</sup> Ricotti, *Villa Adriana*, [2001].

<sup>22</sup> Lancaster, *Concrete Vaulted Construction in Imperial Rome*, [2005].

<sup>23</sup> Heyman, *The Stone Skeleton*, [1997].

<sup>24</sup> Uebliker, *Das Teatro Marittimo in der Villa Hadriana*, [1985].

<sup>25</sup> Mangurian and Ray, *Villa Adriana*.

Before embarking on an analysis of the content of *De architectura*, it is important to understand the basic nature of the text.

Vitruvius never intended for *De architectura* to be a textbook on architecture and engineering. Instead, *De architectura* was intended to be an aide-memoire, a reference manual of compiled architectural knowledge.<sup>26</sup> In the introduction to Book I, Vitruvius writes that *De architectura* is an unoriginal work because he is simply gathering scattered bits of architectural knowledge.<sup>27</sup> However, Vitruvius consciously and unconsciously selects the material that he believed was necessary for an architect to know and shapes the raw information into his own guidelines and suggestions for application. In analyzing and condensing a general body of knowledge, Vitruvius made *De architectura* an original work.

The organization of *De architectura* into ten books was quite deliberate as each book contains very different, though not unconnected, information. In Books I through IV Vitruvius discusses the individual and fundamental components of architecture, such as materials and proportions. In Book V, he shows how the elements are formed into coherent structures and, in Book VI, how the structures interact with the surrounding world. In Book VII, Vitruvius discusses concepts pertaining to the beautification of buildings. Books VIII through X are somewhat incongruous with the previous seven. Books VIII and X are recognizable as engineering guidelines for the transport of water and construction of various machines, respectively. Book IX discusses astronomy and its use in the construction of analemma and clocks.

Only Books I through V will be used in the analysis of Vitruvius in this thesis. These first five books of *De architectura* describe the roots and process which led Vitruvius to form rules for architectural design. The identification of these rules is the key to determining whether they were applied in building construction. The applied rules will then be analyzed for engineering robustness in later chapters.

### 2.1 Books I & II

Book I of *De architectura* is a summary of the fundamental principles of architectural design, construction, and urban planning. It is here that Vitruvius discusses the education of architects. He stresses the importance of a well-rounded education because architecture affects the whole human experience. Beauty, or *venustas*, is the ideal of architecture which can be expressed only by architects who understand the connection between all arts. He also stresses the importance of technical competency, especially the ability to use a ruler and compass. These, Vitruvius writes, are the most fundamental tools of architecture for reasons that become evident in Book III. Technical knowledge, such as geometry and methods of construction, is the channel of *firmitas*, the integrity and strength of a structure. The constraint that unites the practical and ideal natures

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<sup>26</sup> Gros, *Le Projet de Vitruve*, 1.

<sup>27</sup> Vitruvius, *De architectura*, 1.preface.2.

of architecture is *utilitas*, or the concept that beauty and strength must conform to the purpose of the structure so that it will be coherent with respect to its structural elements and its environment.<sup>28</sup>

Book II of *De architectura* is primarily a discussion of materials. Here, Vitruvius provides information on the selection and use of raw materials such as sand, lime, *pozzolana*, and stone.<sup>29</sup> He also discusses conglomerate materials such as brick and concrete as well as the role of timber as reinforcement or use in less glamorous construction.<sup>30</sup> As well as providing practical information about construction materials, Vitruvius discusses the role of materials in *decor*, the concept of appropriate visible beauty, which can be thought of as “architectural etiquette.”<sup>31</sup> Unlike modern materials selection, Vitruvius relies on relative measures of strength and utility of materials. In addition, the actual nature of these materials is largely unknown because of natural variation in the material qualities, variation in origin, and variation of recipes of conglomerate materials. Modern methods of testing and classifying materials involve the destruction of specimen and are therefore undesirable for application to historical buildings.<sup>32</sup> Nonetheless, this section of *De architectura* provides readers with a basic understanding of the materials used in construction during the late Republic and early Roman Empire.

## 2.2 Books III & IV

Books III and IV are the sections which draw the attention of most modern studies of *De architectura*. It is in these two chapters that Vitruvius derives most of his guidelines through philosophical significance and engineering concerns. It is these guidelines that build up to the *firmitas* and *venustas* concepts of architecture.

Columns are explained in detail because they lead up to the basis of Vitruvius’ precepts.<sup>33</sup> In Books III and IV, Vitruvius discusses the origin of Ionic and Doric columns as well as Corinthian capitals.<sup>34</sup> These genres are often considered three separate types of columns, but according to Vitruvius, Corinthian columns are differentiated only in the capital. The shaft and base of the column are either Ionic or Doric as best fits the building.<sup>35</sup> He then derives formula for the correct dimensions of columns based on the diameter, height, and intercolumnar spacing.<sup>36</sup> One of these dimensions is chosen as the metric by which the rest of the building is designed and forms the basis of *distributio* and *symmetria* in buildings. *Distributio* refers to the arrangement of structural elements in a complete design while *symmetria*, not quite meaning symmetry, refers to the proportional balance in the building.<sup>37</sup> *Symmetria* is difficult to characterize on its own and is better understood in conjunction with *eurythmia*, discussed later.

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<sup>28</sup> Vitruvius, *De architectura*, 1.2.

<sup>29</sup> Vitruvius, *De architectura*, 2.

<sup>30</sup> Vitruvius, *De architectura*, 2.9.

<sup>31</sup> Pollio, *Vitruvius on Architecture*, ed. Smith, 23-24.

<sup>32</sup> Moore, “The Pantheon,” <http://www.romanconcrete.com/docs/chapt01/chapt01.htm>.

<sup>33</sup> McEwen, *Vitruvius*, 212-229.

<sup>34</sup> Vitruvius, *De architectura*, 3-4.

<sup>35</sup> Vitruvius, *De architectura*, 4.1.

<sup>36</sup> Vitruvius, *De architectura*, 4.3.

<sup>37</sup> Pollio, *Vitruvius on Architecture*, ed. Smith, 16-18.



This method of basing a design on a single component's dimension is known as modular design. Table 2.1 below summarizes Vitruvius' modular determination of shaft height and intercolumnar spacing as multiples of the base diameter of a column by temple style.

Table 2.1 - Demonstrates the proportional relationships of column diameter, height, and intercolumniation using the base diameter as the standard dimension.

Style	Base Diameter	Shaft Height	Intercolumnar Spacing
<i>araeostylos</i>	1	8	4 or 5
<i>diastylos</i>	1	8.5	3
<i>eustylos</i>	1	9.5	2.25 or 3
<i>pycnostylos</i>	1	10	1.5
<i>systylos</i>	1	9.5	2

Below, in Figure 2.2, is a visual representation of the effect of the modular design described in *De architectura*.

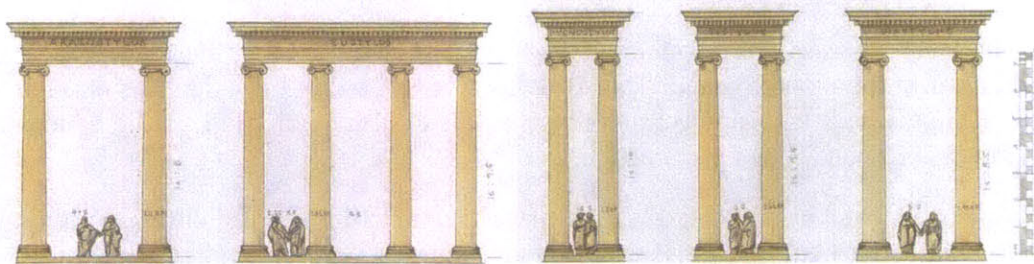


Figure 2.2 - Drawing of the five styles as described by Vitruvius.

Column dimensions also constrain the design of the bases and entablatures. The modular relations for bases are given in Table 2.4 and while the letters correspond to the components of Attic and Ionic type bases as identified in Figure 2.3.

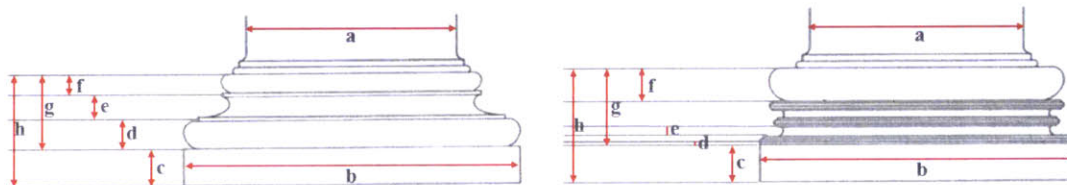


Figure 1.3 - The components of an Attic base (left) and an Ionic base (right). The letters correspond to the relations presented in Table 2.

Table 1.4 - The proportions of column base components as described by Vitruvius.

Component	Proportion	Attic base	Ionic base
a		1	1
b		$\frac{3}{2}$	$\frac{11}{8}$
c		$\frac{1}{6}$	$\frac{1}{6}$
d		$\frac{1}{8}$	$\frac{1}{84}$
e		$\frac{1}{8}$	$\frac{2}{21}$
f		$\frac{1}{12}$	$\frac{1}{7}$
g		$\frac{1}{3}$	$\frac{1}{3}$
h		$\frac{1}{2}$	$\frac{1}{2}$

The breakdown of proportional relations for entablatures can be found in Chapter 3. Figure 2.5 gives a visual representation of the effect of entablature scaling.

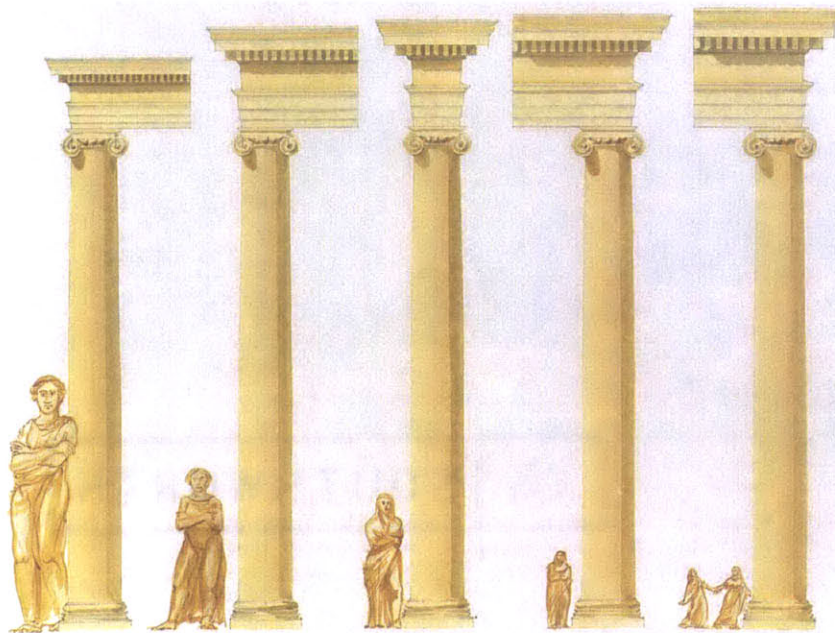
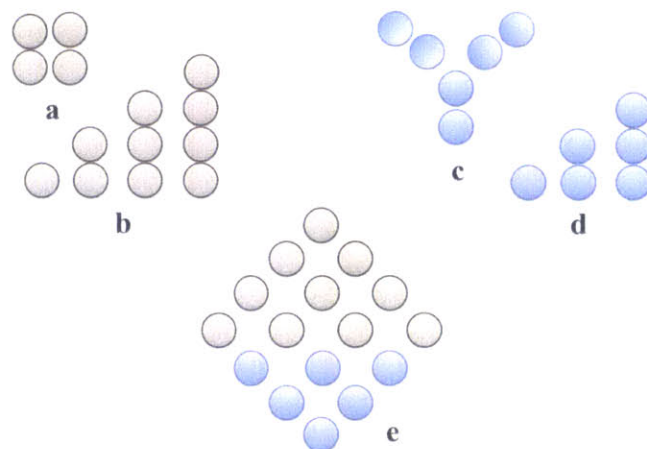


Figure 2 - This drawing shows the entablature scaling for columns of 12-15RF, 15-20RF, 20-25RF, 25-30RF, and 30-40RF, respectively.

Columns and their components are the structural elements which Vitruvius discusses in the most detail. He presents similar formulae for building elements, particularly in temples, including the pronaos, cella, and entrance.<sup>38</sup> Vitruvius justifies these rules in two ways. First, he states that the rules protect against structural failure. For example, limiting the height of a column based on a relationship with the column diameter prevents the column from being built so tall that it crumbles under its own weight.<sup>39</sup> Second, he explains the presence of proportions as a natural phenomenon. Circles and squares had great significance in ancient times. By association, because rulers are necessary to draw squares and compasses are necessary to draw circles, they are the essential tools of architecture. These two tools are essential to architecture, because they are the tools of building layouts, or *ichnographia*.<sup>40</sup> The sum of *ichnographia*, *distributio*, *symmetria*, and *decor*, is the *dispositio*, a term that refers to the entirety of a building's design.<sup>41</sup>

The sphere, a symbol of the universe, was observable to man as the circular horizon, as written by Cicero.<sup>42</sup> It represents unity, the first fundamental number of Roman philosophy. The square has four points and four sides; four is the elemental number of the universe in ancient philosophy because there are four cardinal directions, four elements, and other naturally occurring sets of four. Another significant number, six, is a tripartite number, an echo of tripartite gods. The factors of four (one, two, three, and four) add up to ten, the number of perfection, according to Vitruvius.<sup>43</sup> Ten, when added to six, creates another perfect square. This is presented using counters in the figure below.



**Figure 2.6 - Using counters, the numbers four (a) and six (c) are broken into components in (b) and (d), respectively. The configuration in (e) shows the return to a squared factor.**

<sup>38</sup> Vitruvius, *De architectura*, 4.3.

<sup>39</sup> Vitruvius, *De architectura*, 3.3.

<sup>40</sup> Pollio, *Vitruvius on Architecture*, ed. Smith, 23.

<sup>41</sup> Pollio, *Vitruvius on Architecture*, ed. Smith, 20.

<sup>42</sup> Cicero, *De natura decorum*, 2.47.

<sup>43</sup> McEwen, *Vitruvius*, 39-54.



Vitruvius shows that the connection of these numbers, four and ten, and these shapes, the circle and square, is evident in the proportions of man.<sup>44</sup> Man has four limbs, ten digits on his arms, and ten digits on his legs. When a man is laid out, his limbs mark the four points of a perfect square showing connection with the fundamentalism of the number four.<sup>45</sup> Furthermore, a circle centered at the man's navel will perfectly circumscribe the man and square as well as touch the ten digits of the hands and feet and signifies the connection of the proportions as perfect and complete like the universe. This is man's *ratio* or coherence.<sup>46</sup>

However, it is important to note that Vitruvius warns against adhering strictly to the proportions of man.<sup>47</sup> He writes that beauty in architecture must be visible to the casual eye. Just as human beauty exists despite variation in proportion, architectural proportions can be tweaked to comply with aesthetic coherence. The aesthetic coherence is known as *eurythmia*. To improve *eurythmia*, *dispositio* takes precedence over the geometry of architectural design, or *ordinatio*.<sup>48</sup> Vitruvius went to great lengths to avoid having to draw images because he believed it would signify weak communication skills, but the image of this Vitruvian man is easily recognized thanks to Leonardo Da Vinci's drawing, shown below.<sup>49</sup>

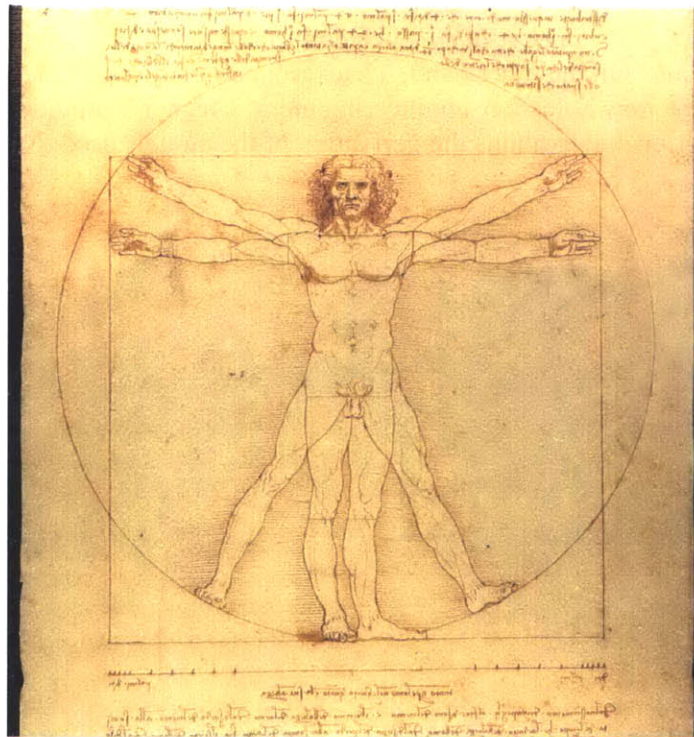


Figure 2.7 - Leonardo da Vinci's Vitruvian Man.

<sup>44</sup> Vitruvius, *De architectura*, 3.1.

<sup>45</sup> McEwen, *Vitruvius*, 39-54.

<sup>46</sup> McEwen, *Vitruvius*, 156-182.

<sup>47</sup> Vitruvius, *De architectura*, 3.1.

<sup>48</sup> Pollio, *Vitruvius on Architecture*, ed. Smith, 20-21.

<sup>49</sup> Vitruvius, *De architectura*, 1.1.

## 2.3 Book V

The last chapter of *De architectura* that is relevant to architectural design is Book V. In this chapter, Vitruvius shows how the fundamental concepts and rules from the previous chapters are put together to form a whole, the final structure. While Books III and IV dealt with design for strength and design for beauty, Book V stresses design for functionality in the pursuit of *utilitas*. In other words, a structure must exhibit internal coherence as well as external coherence. External coherence depends on the environment of the building and the original purpose of the building.<sup>50</sup> For example, a temple requires different design principles of *venustas* and *utilitas* than a forum. Structure determines the type of interactions that are possible in a building. For example, colonnades encourage interactions that involve walking.<sup>51</sup> A central platform allows discussion with a wider audience while a stage against a wall is more appropriate for performances and speeches. Finally, Vitruvius discusses how the layout of a building will affect the acoustics and how to use structural elements as a tool in manipulating acoustics.<sup>52</sup>

This concludes the summary of the portions of *De architectura* that are relevant to architecture and construction. The analysis of Vitruvius proposed in this thesis requires two frames of reference. In the discussion of relevance, it is necessary to identify the axioms to be studied while keeping in mind the contextual reasoning that led to their creation. Chapter 2 provides a brief summary of *De architectura* and a condensed interpretation of the architectural tenets written by Vitruvius. On the other hand, a background of structural analysis is needed to judge whether *De architectura* holds any engineering merit. Chapter 3 provides general information on construction and explains the derivation of the models used to quantify Vitruvian engineering.

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<sup>50</sup> McEwen, *Vitruvius*, 198-201.

<sup>51</sup> Vitruvius, *De architectura*, 4.9.

<sup>52</sup> Vitruvius, *De architectura*, 4.4-4.8.

The previous chapter identified the precepts which would indicate Vitruvian influences in a building or structure. It set the foundation for the qualitative analysis of buildings. To complete the analytical objectives, analytical models must be formed to provide quantitative points of comparison for a structural evaluation. This chapter describes the sources and process which led to the final models.

### 3.1 Materials

Structural mechanics relies heavily on the properties of the materials used in construction. In linguistics, the meaning of a sentence is derived from the meaning of the words and the order in which they are uttered. Similarly, the strength of a structure is determined by the strength of the materials used and by the configurations of those materials into structural elements. Vitruvius understood this relationship, and expanded upon it. He states that a building's coherence is not limited to the coherence of its individual elements, but also includes the building's coherence with its environment.<sup>53</sup> Therefore, to begin to understand the designs of ancient buildings, it is necessary first to identify the materials available for construction and the constraints imposed by material choice.

Vitruvius gives brief descriptions of the materials available to his contemporary architects.<sup>54</sup> However, modern structural analysis requires a more quantitative knowledge of those materials, especially those used in the structures of interest. These materials are limestone and marble, as well as bricks and concrete.

The most exotic, and expensive, of these masonry materials was marble. In ancient times, marble had to be transported from quarries around Italy, in Greece, Africa, and Asia Minor. Both Augustus and Hadrian favored *marmor lunese*, Luna marble, which due to its fine grain texture is easy to work with for more detailed stonework.<sup>55</sup> This was the marble that Suetonius refers to when he writes that Augustus found Rome "built of brick and left it in marble."<sup>56</sup> Marble is a type of metamorphic rock and can refer to metamorphosed or unmetamorphosed limestone varieties, though Luna marble is of the metamorphosed type. It has a higher uniaxial compressive strength that makes it more favorable for structural use as opposed to the less strong, more colorful varieties which are used as facing materials.<sup>57</sup> An example of a Luna marble frieze is presented below in Figure 3.1.

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<sup>53</sup> McEwen, *Vitruvius*, 198-201.

<sup>54</sup> Vitruvius, *De architectura*, 2.

<sup>55</sup> Malacrino, *Constructing the Ancient World*, translated by Hyams, 24.

<sup>56</sup> Suetonius, *Augustus*, 28.

<sup>57</sup> Winkler, *Stone in Architecture*, pp. 45.





**Figure 3.1 - A Luna marble frieze from the Basilica of Neptune behind the Pantheon in Rome.**

Limestone is a sedimentary rock, thus the strength of the rock is dependent on the compaction force which created it.<sup>58</sup> It is quite common around Italy and it is easy to cut limestone into blocks or to carve it. For these reasons, it was a popular construction material in the ancient world. Limestone was also commonly used as an aggregate in filled walls and some varieties of ancient concrete.<sup>59</sup> It has a high solubility which requires modern architects to be cautious about exposure or intrusion of water to limestone structures, especially in locations prone to acid rain. However, it is only in modern times that the solubility of limestone has been studied. Historically, limestone was used in construction as large blocks, especially in Greece.<sup>60</sup> Construction of this type, *opus quadratum*, was most commonly used for foundations, walls, entablatures, and arches.<sup>61</sup>

The third material of interest is Roman concrete, *opus caementicium*. Ancient Roman concrete was nothing like modern concrete.<sup>62</sup> While modern concrete is poured into frames and molds, Roman concrete was laid into frames in layers, similar to brick and mortar. *Caementa* were unshaped and unworked stones, analogous to bricks, which were laid in and covered with mortar. This can be seen in the picture of *opus caementicium* in Figure 3.2. *Caementa* did not refer to a single type of rock, as shown by the varieties of travertine and tufa present in ancient buildings. These rough stones were selected on an availability basis, and later, for certain properties. For example, early Augustan buildings used a single type and size of stone throughout the entire building.<sup>63</sup> The construction of the Pantheon, and other vaulted structures, required more thought with regard to the weight of the material. Thus, the *caementa* was graded in density with heavier stones in the foundations and abutments while less dense stones were used towards the top of the structure. The mortar used in Roman construction was most often made with *pozzolana*, light volcanic ejecta.<sup>64</sup> Its consistency ranged from fine ash to small walnut-sized pieces. Four examples of *pozzolana* are shown in Figure 3.2. The *pozzolana* was mixed with

<sup>58</sup> Winkler, *Stone in Architecture*, pp. 45.

<sup>59</sup> Malacrino, *Constructing the Ancient World*, translated by Hyams, 13.

<sup>60</sup> Malacrino, *Constructing the Ancient World*, translated by Hyams, 106-107.

<sup>61</sup> Malacrino, *Constructing the Ancient World*, translated by Hyams, 111-114.

<sup>62</sup> Lancaster, *Concrete Vaulted Construction in Imperial Rome*, 3.

<sup>63</sup> Lancaster, *Concrete Vaulted Construction in Imperial Rome*, 59.

<sup>64</sup> Lancaster, *Concrete Vaulted Construction in Imperial Rome*, 54-55.

sand and lime, and sometimes other materials, to create a mortar that was notably stronger than lime mortars.<sup>65</sup> The flexibility of the materials in Roman concrete made it favorable for just about any structural element, from foundations, to walls and columns, to vaults and domes.



Figure 3.2 - On the left, an example of *opus caementicium*. On the right, four types of *pozzolana*.

Brick construction was common in ancient Rome, especially before the reign of Augustus.<sup>66</sup> Roman bricks were an improvement on the previously popular mud bricks; they were made of mostly clay because, as Vitruvius wrote, soil bricks could not withstand moisture.<sup>67</sup> Other aggregates, especially other sedimentary rubble, were added to the clay so that as the bricks dried, they would not crack or settle.<sup>68</sup> Vitruvius had many cautionary words on making bricks; he described when and how they should be dried, how they should be used, and explained the dangers of using wet bricks in construction.<sup>69</sup> For most civic buildings, brick was avoided in favor of stone, and when it was used, whether for walls or columns, it was always covered with stucco or other facings.<sup>70</sup>

The material properties used for the models in this paper were estimated on the basis of three sources: previous studies of structures, geological surveys, and modern replication experiments. First the dominant material used in the construction of each structural element (e.g. column, wall, arch, etc.) needed to be identified. In some cases, such as the bricks use in Hadrian's Villa, the material had been previously identified and studied by other scholars. In other cases, like the Teatro Marittimo, only the quarries of origin were known and geological

<sup>65</sup> Lancaster, *Concrete Vaulted Construction in Imperial Rome*, 55-56.

<sup>66</sup> Lancaster, *Concrete Vaulted Construction in Imperial Rome*, 51-52.

<sup>67</sup> Vitruvius, *De architectura*, 2.3

<sup>68</sup> Malacrino, *Constructing the Ancient World*, translated by Hyams, 48-49.

<sup>69</sup> Vitruvius, *De architectura*, 2.3

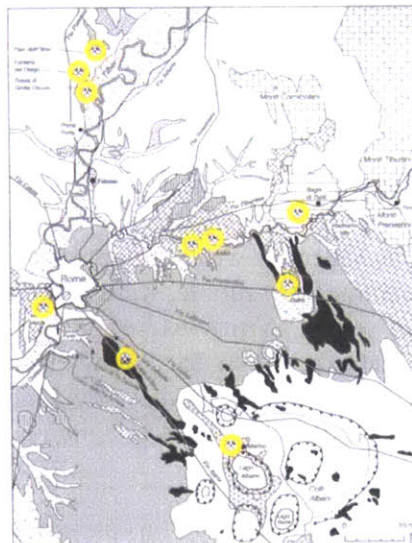
<sup>70</sup> Malacrino, *Constructing the Ancient World*, translated by Hyams, 73.



surveys were consulted to determine the type of rock that was most likely used. Next, the material properties were determined, primarily through the use of several material property tables from engineering and geology texts. Ancient conglomerates such as concrete and mortar have been replicated and the results of stress-strain experiments yielded approximate material properties.<sup>71</sup>

### 3.2 Masonry

Masonry architecture offers many advantages over other types of construction. As was the case for most pre-industrial cities, ancient Rome was highly susceptible to fires due to the high density of homes and poor urban organization. Some, like the fire of 64 AD, managed to spread quickly and caused mass destruction of many quarters. Therefore, one of the attractive qualities of masonry was its non-combustible nature, though it was generally only affordable for civic buildings and the private estates of the wealthy. The immediate area of Rome had relatively few local quarries that produced stones of the desired strengths, sizes, or types. The map below shows the location of eight ancient quarries, including the quarry near Hadrian's Villa at Tivoli.



**Figure 3.3 - A map of Rome highlighting the locations of ancient quarries.**

The most significant advantage that masonry offers is strength. Stones used in masonry generally have high uniaxial, unconfined, compressive strengths.<sup>72</sup> The limitations of masonry are generally related to the difficulty of shaping stones into the desired shapes and dealing with the weight during transport and construction.<sup>73</sup> Working with stone required skilled laborers for

<sup>71</sup> Moore, "The Pantheon," <http://www.romanconcrete.com/docs/chapt01/chapt01.htm>.

<sup>72</sup> Heyman, *The Stone Skeleton*, 12.

<sup>73</sup> Malacrino, *Constructing the Ancient World*, translated by Hyams, 40.

the technical tasks and an abundance of strong labor for transport and construction.<sup>74</sup> In addition, because quarries were often located across the Mediterranean Sea, there were long delays and high costs associated with acquiring materials.<sup>75</sup> Finally, as Vitruvius himself describes in Book X, the hoisting machines involved in building construction required almost as much skill and labor to build.

Modern studies of masonry architecture have become increasingly complicated; but with the help of certain assumptions and boundary conditions, the mechanics can be simplified. Two assumptions can be made about the materials used in masonry structures: that they possess infinite compressive strength and have zero tensile strength.<sup>76</sup> Any geologist will immediately object to these two assumptions, and rightly so.

Depending on the geological processes that formed the rock in question, stone can possess exceptional compressive strength.<sup>77</sup> Given that masons chose stone types for their strength, the assumption of infinite strength is reasonable but should still be checked. Infinite compressive strength should also not be attributed to filled walls- walls that consist of two 'shells' filled with compacted material. This is because these types of walls experience slump over time due to the variations in moisture and pressure that can cause a ratcheting effect that drives the shells apart and causes uneven load distribution.<sup>78</sup> The slump and drift effect is illustrated in Figure 3.4.

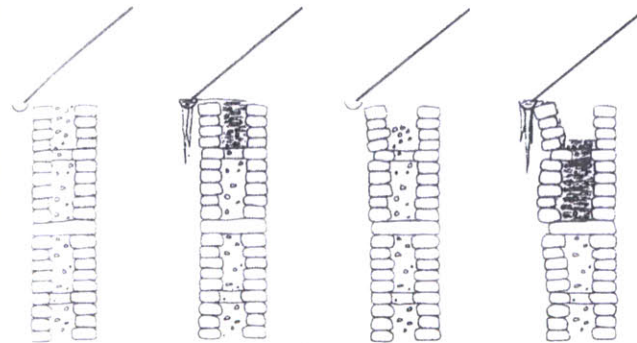


Figure 3.4 - The drifting and slumping of filled masonry walls.

The assumption of zero tensile strength should be taken cautiously. This assumption does not describe the internal tensile strength to stone but rather applies to the material used to join stones. Even the joining material, such as mortar, will possess some measure of tensile strength, but will be significantly lower than the tensile strength of stone.<sup>79</sup> Assuming zero tensile strength is a conservative assumption that will provide the lower bound of the overall structure's strength.

<sup>74</sup> Heyman, *The Stone Skeleton*, 2.

<sup>75</sup> Malacrino, *Constructing the Ancient World*, translated by Hyams, 25.

<sup>76</sup> Heyman, *The Stone Skeleton*, 14.

<sup>77</sup> Winkler, *Stone in Architecture*, pp. 45.

<sup>78</sup> Heyman, *The Stone Skeleton*, 84.

<sup>79</sup> Heyman, *The Stone Skeleton*, 14.

Finally, the third assumption in masonry models is the no-slip condition.<sup>80</sup> This is generally true for stone, especially in ancient times where the methods used to cut and shape stone left a rough enough surface to prevent slip. In addition, it was common practice to further roughen the joined surfaces of the stone blocks by chiseling into them.<sup>81</sup> This assumption can only be applied to stone-stone interfaces (including stone-mortar interfaces) and not to stone-wood interfaces in structures reinforced with timber.<sup>82</sup> It can also be noted that timber reinforcements were only effective in lengths of up to 1 meter because of the surface interactions with the stone shells and fill materials.<sup>83</sup>

### 3.3 Mechanics

The structural analysis models used for this investigation required the derivation of equilibrium equations to account for force distribution in structural elements. For the sake of simplicity, the models evaluate the stability and strength of individual elements, e.g. walls and columns, rather than trying to characterize the interactions of an entire building. This approach was especially helpful for the study of ancient structures because very few are preserved in their entirety.

Each of the models begins by identifying the forces acting on an element about the center of gravity and about the attachment points. Through force-body diagrams, the forces were combined to determine the total load experienced by a single element. Using material properties such as Young's modulus and the dimensions of an element, the inherent strength and stability was calculated. The actual load is then compared with the calculated maximum load to determine the stability and safety of the structure. Attachment points of foundations with other structures were modeled as simple, fixed points of attachments because data for recessed attachments is unavailable.

Ancient Greek and Roman foundations have been studied extensively and were therefore not analyzed in the following models.

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<sup>80</sup> Heyman, *The Stone Skeleton*, 14.

<sup>81</sup> Malacrino, *Constructing the Ancient World*, translated by Hyams, 38.

<sup>82</sup> Heyman, *The Stone Skeleton*, 14.

<sup>83</sup> Heyman, *The Stone Skeleton*, 14.



### 3.3.1 Columns

The analysis of columns performed here, modeled them as cylindrical rods of uniform diameter as shown in the figure below:



Figure 3.5 - Loading of a fixed-fixed end column.

Columns can be fluted, but the flute depth was much smaller than the diameter of the column. They also did not usually have uniform diameter. The base diameter was generally larger than the diameter of the top, more so with Ionic columns than Doric columns. In addition, the bases of Ionic columns were carved into rings of varying diameter.<sup>84</sup> Assuming a uniform diameter allows conservative calculations of strength and load distribution.

The MATLAB code *estimate\_columnar\_load* estimates the load borne by a single column based on the dimensions of the column. The diameter and height of the column determined the maximum heights of the entablature elements while the intercolumnar spacing was used to calculate the volume of the entablature that rested on a single column. The density of the entablature material was then used to calculate the mass of material resting on a single column. The Vitruvian proportional rules for column entablature elements is summarized in Table 3.6.

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<sup>84</sup> Pollio, *Vitruvius on Architecture*, ed. Smith, 25.

Table 2.6 - This table shows the proportional dimensions of the entablature elements as multiples of the base diameter. Please note that unlike the previous tables, the module of the diameter is given as 100.

Feature	Height of Columns			
	12-15RF	15-20RF	20-25RF	25-30RF
Column Base Diameter	100	100	100	100
Total Architrave Height	57.1	79.1	82.3	85.7
Total Fascia Height	42.8	59.3	62.0	64.3
Total Frieze Height (unadorned)	42.8	59.3	62.0	64.3
Total Frieze Height (adorned)	71.4	98.9	102.9	107.1
Total Dentil Height	16.7	23.1	24.0	25.0
Total Cornice Height	14.3	19.8	20.6	21.4
Total Entablature Width	7.1	9.9	10.3	10.7

Column stability was analyzed with respect to uniaxial loads along the height and moments about the base. The primary modes of column failure are buckling and tilting. The MATLAB code *column\_buckling* finds the critical loads in buckling and tilting failures.

Buckling occurs when the load exceeds the axial strength of the column and causes the column to deform, as shown in Figure 3.7.



Figure 3.7 - Buckled configuration of a fixed-fixed end column.

Buckling will originate at the weakest point in the column, which can be at a point farthest from the supports or at a point where the material has anomalous microstructure. The stress and strain at the site of deformation will propagate until they exceed the tensile, compressive, or shear strength of the material. There exist two models of buckling loads: the Euler buckling model and the elastic buckling model.<sup>85</sup> The elastic buckling model uses an effective length, measured as in the figure below, to qualify the load as calculated by the Euler model of buckling.<sup>86</sup> The dimensions of the column that are needed in buckling calculations are

<sup>85</sup> Craig, *Mechanics of Materials*, 616.

<sup>86</sup> Craig, *Mechanics of Materials*, 616.

the shaft length,  $L$ , and the base radius of the column,  $R$ . For simplicity,  $K$ , the effective qualifier was be assumed to equal 1; thus effective slenderness ratio,  $L/R$  is the same for both models.

Thus the critical compressive stress is given by the equation:

$$\sigma_{cr} = \frac{\pi^2 E}{(L/R)^2} \quad (1)$$

and the critical load is given by the equation:

$$P_{cr} = \frac{\pi^2 EI}{L^2} \quad (2)$$

In Equations 1 and 2 above,  $E$  is the Young's Modulus of the material and  $I$  is the moment of inertia given in Equation 3 below.

$$I = \frac{\pi R^4}{4} \quad (3)$$

Using the critical compressive stress, the self-crushing height,  $h_{crush}$ , of the column can be calculated as well.

$$h_{crush} = \frac{\sigma_{cr}}{\rho g} \quad (4)$$

The density of the material is found in materials property data tables. Failure by tilting requires looking at the moments a column will experience. The equilibrium moment about the center of the base is given by the equation:

$$M = k_{\theta} \theta \quad (5)$$

In Equation 5,  $k_{\theta}$ , is the resistance to angular displacement. At some displacement  $\theta$ , the column will experience a moment:

$$M = PL \sin \theta \quad (6)$$

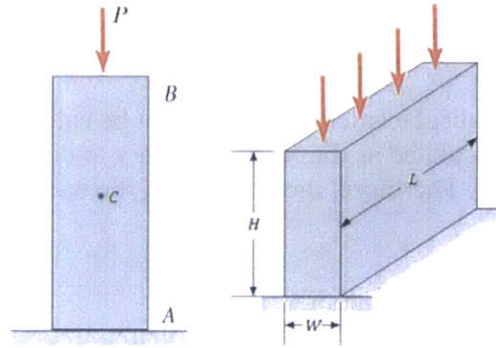
This moment is due to the axial load,  $P$ , on the column. The moment balance at neutral equilibrium marks the critical moment between stable equilibrium and unstable equilibrium. Thus the critical tilting load (at small displacements) is given by:

$$P_{tilt} = \frac{k_{\theta}}{L} \quad (7)$$

However,  $k_{\theta}$  is not easily calculated. It depends on the geometry of the base of the column and there is too much variability in the geometry to calculate a generic  $k_{\theta}$ . The variability is present in the type of base as well as in the individual components (see Figure 2.3) of each base. Therefore, the analysis of columns will not include failure by tilting.

### 3.3.2 Walls

For simplicity, only solid walls were chosen so that the ratcheting mechanism of failure did not need to be modeled. Therefore, walls were modeled as a flat slab of uniform thickness,  $W$ , as shown in Figure 3.8.<sup>87</sup> This flat slab was characterized as having uniform unconfined compressive strength because the volume of rock is much greater than the volume of mortar. Walls of this nature fail in similar modes as columns, buckling and tilting. The MATLAB code *wall\_load* estimates the critical loads at which failure occurs for both modes. The primary modes of wall failure are buckling and destabilization.



The buckling load and stress were found using the same method as for columns with the significant difference being that the inertial moment of a wall was calculated as the inertial moment of a rectangular slab.

Thus the critical compressive stress is given by the equation:

$$\sigma_{cr} = \frac{\pi^2 EI}{AL^2} \quad (8)$$

Figure 3.8 - Wall modeled as a slab of uniform thickness with load distributed along the top.

and the critical load is given by the equation:

$$P_{cr} = \frac{\pi^2 EI}{L^2} \quad (9)$$

<sup>87</sup> Heyman, *The Stone Skeleton*, 84.

The moment of inertia is found through the equation:

$$I = \frac{1}{12}m(L^2 + W^2) \quad (10)$$

Destabilization occurs when a load pushes the center of gravity beyond the boundaries of the base while deformation requires a moment to exceed the stabilizing moment about the center of gravity. Thus the critical tilting load, for small displacements, is given by:

$$P_{tilt} = \frac{k_{\theta}WL}{H} \quad (11)$$

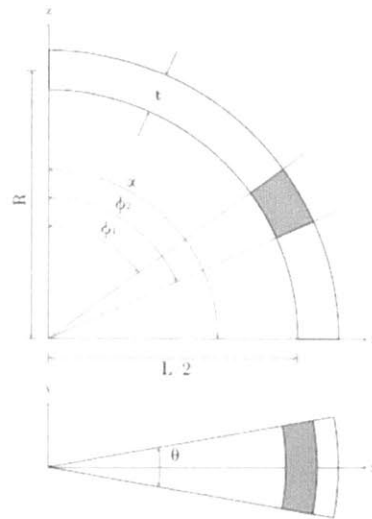
Again, the resistance to angular displacement,  $k_{\theta}$ , could not be calculated easily for walls. However, the tilt can also be calculated as the force necessary to displace the wall's center of gravity past the limit of the base. Therefore, the tilt force of the wall was calculated as:

$$P_{tilt} = \frac{mgW}{2} \quad (12)$$

### 3.3.3 Vaulted Structures

The analysis of vaulted structures had two objectives: to confirm stability by finding an equilibrium-state using thrust line analysis and to calculate the total load that would be placed on supporting elements such as walls and columns. By the master safe theorem, as long as one equilibrium state can be found, a structure can be stable.<sup>88</sup>

The MATLAB code *dome\_thrust\_line* estimates the weight of a dome or vault, the thrust force, and confirms that at least one stable thrust line exists. This program is a modified version of the program used by Jennifer Zessin to find a minimum thrust line.<sup>89</sup> For this investigation, any thrust line within the boundaries of the abutments was acceptable and the code was modified to accept vaults as well as domes. The code requires the following dimensions of a dome: the radius,  $R$ , span,  $L$ , angle of embrace,  $\alpha$ , and thickness,  $t$ . These dimensions are shown below.

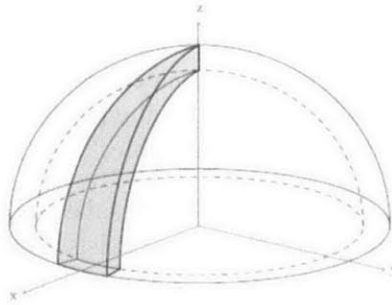


**Figure 3.9 - Diagram of the measurements of a dome's projection.**

Using these dimensions, the dome is characterized by uniform lunes. A lune is the segment of a dome bounded by two intersecting planes at the center of the dome. The angle between the two planes is identified as  $\theta$ .

<sup>88</sup> Heyman, *The Stone Skeleton*, 8.

<sup>89</sup> Zessin, *Collapse Analysis of Concrete and Masonry Domes*, 41-50.



**Figure 3.10 - A lune is a segment of a dome bounded by two intersecting planes.**

The program first calculates the volume and weight of a single lune and the full structure. Then the programs calculate the boundaries of the thrust line at the crown and base of the dome to set the true/false condition of equilibrium and find the resultant thrust force. For an excellent, and very thorough, explanation of dome thrust line analysis, Lancaster provides step-by-step instructions in her book.<sup>90</sup>



### 3.3.4 Model Limitations

The models presented here provide approximate models of the structural behavior of columns, walls, and domes. The results are conservative and, as will be shown, are comparable to the results of more advanced models.

Nevertheless, there are three main limits on the accuracy of the models which must be discussed. The first limit is imposed by material properties. This source of error is common to most modern studies of ancient structures. Ancient materials are limited in availability and fiercely protected. Until materials testing has progressed to the point where it does not destroy samples, ancient materials must be replicated before they can be studied.

The second limit is due to the simplifying assumptions made throughout the derivations above. For the scope of this thesis, the assumptions were necessary but more advanced models will provide better characterization of structural element behavior. For example, future models for column analysis should be able to model the base geometry, the effect of fluted shafts, and the varying shaft diameters. For walls, failure models should be able to account for the joining material and construction of the wall. An entire study could focus on determining how to calculate  $k_\theta$  for various structural elements.

Finally the greatest limitation of these models is that they cannot account for changes in stability that are due to external influences. These include vibrations, varying moisture in the surrounding soil, and the weathering effects on the strength of rock.

For the scope of this thesis, the models provide sufficient information for the objectives of the analysis. Now, having identified the rules set in *De architectura* and formed models to provide numerical points of comparison, the analysis of actual structures can begin.

## VITRUVIUS IN THE AGE OF AUGUSTUS

The two preceding chapters have established the background knowledge by which to analyze the relevance and merit of Vitruvian architectural principles. Vitruvius wrote *De architectura* near the beginning of Augustus' imperial reign. Therefore, the analysis of an Augustan building will show whether Vitruvius had any immediate relevance and or impact on architecture.

### 4.1 The Temple of Divus Julius

The Temple of Divus Julius, also known as the Temple of the Deified Julius Caesar was dedicated by Augustus in 29 BC in honor of his adoptive father, Julius Caesar. It stood in the Forum Romanum off the Via Sacra facing the Temple Concordia.<sup>91</sup> Its location among contemporary structures can be seen below.

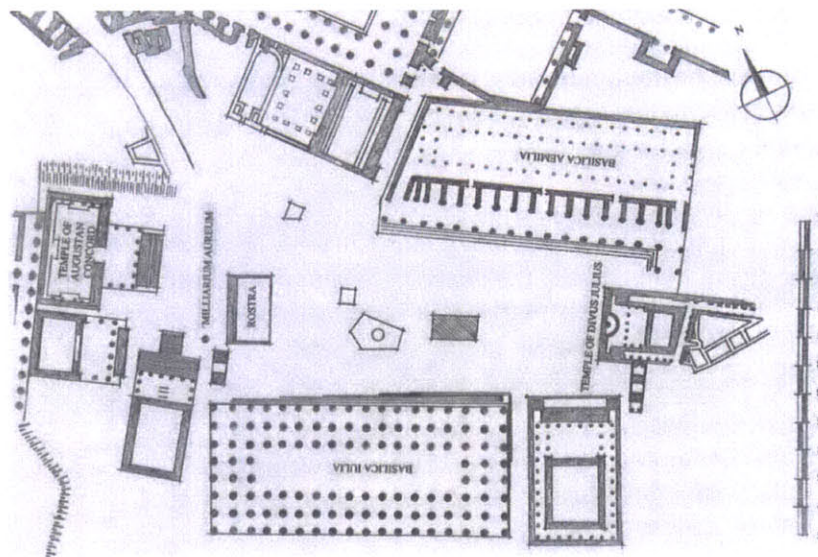


Figure 4.1 - The Forum Romanum in the second century BC.

Not much remains of the structure today. Descriptions have been gleaned from literary sources, including Vitruvius, and coins (such as the one in Figure 4.2), in order to form possible reconstructions. One such reconstruction is shown in Figure 4.3. In addition, there are no surviving contemporary maps or plans that include the temple. The Severan marble, which contains a map of the Roman Forum, is missing a piece where the Temple of Divus Julius would be located. Gros has dedicated an enormous amount of effort to reconstructing the original structure of the temple and compare the architectural style to the rules presented in *De architectura*.

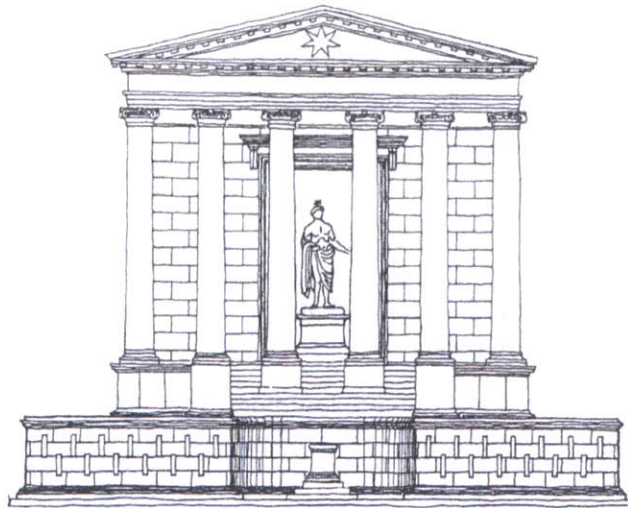
<sup>91</sup> McEwen, *Vitruvius*, 175.



**Figure 4.2 - A coin (aureus) from 36 BC with the head of Octavian on one side and the Temple of Divus Julius on the other side.**

In particular, he was able to quantify Vitruvius' rules concerning the proportions of columns and their elements. Though the columns themselves do not remain, Gros has been able to conclude that the columns of the Temple of Divus Julius almost certainly adhered to Vitruvian proportions and were possibly even the original source. To get to this conclusion, Gros had to form possible reconstructions of the temple based on literary evidence and images from coins. In addition, he conducted similar research for a wide sample of Augustan buildings to evaluate the strength of his conclusion against a large set of structures.

Descriptions of the capitals in the Temple of Divus Julius vary, though it seems likely that they were Corinthian capitals. This, too, complies with Vitruvius' belief that Corinthian capitals symbolized victory and the suggestion that they be used only in certain contexts.<sup>92</sup>



**Figure 4.3 - A drawing of a possible reconstruction of the Temple of Divus Julius.**

<sup>92</sup> Gros, *L'Architecture Romaine*, 122-143.

As of yet, there are no sources which describe the measurements of the entablatures besides *De architectura*. Therefore, application of Vitruvian moduli cannot be quantitatively confirmed.

Further evidence that the Temple of Divus Julius may have been a source for Vitruvius' guidelines comes from studying the squared nature of the temple. Squaring refers to the orientation of buildings, especially those where augurs took the auspices, with regards to the four cardinal directions. Though Vitruvius asserts that temples must face west and altars must face east, Gros has found that Augustan temples faced in all sorts of directions.<sup>93</sup> However, the Temple of Divus Julius alone faces west. The orientation is purposeful; the rising sun would be at the back of an augur and the setting sun would have illuminated the giant statue of Julius Caesar in the cella.<sup>94</sup> While at the time that *De architectura* was written, temples were oriented in various ways, it seems plausible that given the increasing importance of the imperial cult and the significance which Vitruvius stresses regarding temple/altar orientation, later west-oriented temples were based on *De architectura*.

## 4.2 Conclusions

The evidence found by Gros supports the conclusion that Vitruvius was not as disconnected from the actual state of architecture as is sometimes argued. At the same time, Gros has found that Vitruvius rules were not absolute; they were not applied consistently. This may be due to lack of awareness of *De architectura*. The dissemination of texts in ancient times was much slower than it is today. It may also be the case that Vitruvius did not yet possess sufficient *autoritas* for other architects to follow his rules. Still another possibility is that *De architectura* was written too late to impact Augustus' architectural practices. Whatever the case may be, *De architectura* did have some relevance to Augustan architecture. It is even possible that Augustan architecture was the source of some of Vitruvius' rules. Without an explanation from Vitruvius, it is impossible to determine the true sources for the rules he presents.

Gros' conclusions can be nicely summarized when considering the concept of squaring buildings. Vitruvius dedicated an entire section of *De architectura* to explaining what directions temples and altars should face. Yet, the only Augustan building which adheres to these rules is the Temple of Divus Julius. In fact, Augusta buildings face just about every direction. Later, however, new temples were constructed facing west more often than not. This may be due to the architects wanting to imitate Augustus' building, though in that case, they could have faced any direction. It is more likely that given time, *De architectura* gained a wider audience and this was reflected in the standardization of architecture.

This hypothesis can be tested by studying buildings from later imperial reigns. Thus, the next section will study Hadrian-era buildings to determine whether Vitruvius' work continued to be relevant to Roman architecture.

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<sup>93</sup> McEwen, *Vitruvius*, 175.

<sup>94</sup> McEwen, *Vitruvius*, 175-177.



## VITRUVIUS IN THE AGE OF HADRIAN

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In this chapter, Hadrianic buildings will be studied to determine whether *De architectura* was relevant despite the length of time.

### 5.1 The Pantheon

The Pantheon is one of the most well-known ancient Roman buildings. The original Pantheon was built in 27 BC by Marcus Agrippa but was destroyed in the Great Fire of 80 AD. It was then rebuilt by Domitian and destroyed once again in 110 AD. The final reconstruction finished by Hadrian in 126 AD, amounted to an entirely new structure and was preserved because it was consecrated as a church in the 7th century AD.<sup>95</sup> A reconstruction and recent picture are presented below.

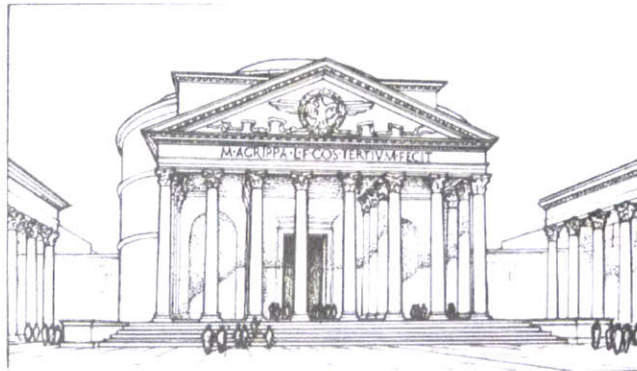


Figure 5.1 - A view of the Pantheon as reconstructed by Hadrian.



Figure 5.2 - A picture of the Pantheon amidst modern-day Rome.

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<sup>95</sup> Rivoira, *Roman Architecture and its Principles of Construction under the Empire*, 118-159.

The original Pantheon was built in the lifetime of Vitruvius, yet he never mentions it in *De architectura*. The lack of content concerning vaulted construction, as well as other ignored designs, is often cited as evidence of Vitruvius' disconnection with the actual state of Roman architecture.<sup>96</sup> Some scholars, like Gros, argue that Vitruvius may simply have wanted *De architectura* to reflect the simplicity of structural design.<sup>97</sup> Whatever the case may be, the Pantheon's current structure shows some traces of Vitruvian influence in the column design.

The predominant geometry is Euclidean, involving the use of circles to form concave and convex rooms.<sup>98</sup> However, the columns in the portico are thought to have been kept in the same style as those of the original Agrippan structure.<sup>99</sup> Through the perspective of Vitruvius, the columns are Ionic in base with Corinthian capitals. The use of Corinthian capitals, if deliberately in keeping with Vitruvian principles, is predictable using the reasoning and symbolism of *De architectura*. Corinthian capitals, according to Vitruvius, are primarily symbols of victory. The Pantheon was dedicated after Augustus' triple triumph and after he became emperor. It is appropriate, then, that Marcus Agrippa, a longtime friend of Augustus, would have chosen to use Corinthian capitals to echo the victories of Augustus.<sup>100</sup> The shaft height of the current columns does not match the height that Vitruvius predicts using the base diameters. However, as Moore notes, the new shafts were shorter because of the quarry capacity, the distance over which they had to be transported, and the manner in which they had to be transported. Despite the heights being short, the intercolumnar spacing, entablature height, and pediment height all follow the formula set by Vitruvius.<sup>101</sup>

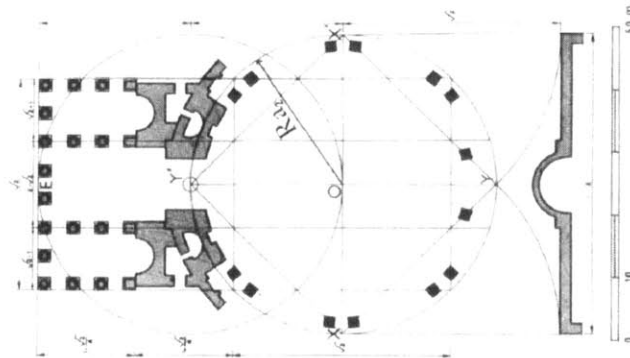


Figure 5.3 - One of the proposed Euclidean geometry based ground plans.

The spherical dome of the Pantheon is the world's largest unreinforced concrete dome and has therefore been studied extensively. Scholars have investigated whether the stability is due to the design or the materials. Lancaster applied thrust line analyses to study the way hoop stresses and meridional stresses interact to form a resultant thrust force at the abutment of the

<sup>96</sup> McEwen, *Vitruvius*, 2.

<sup>97</sup> McEwen, *Vitruvius*, 3.

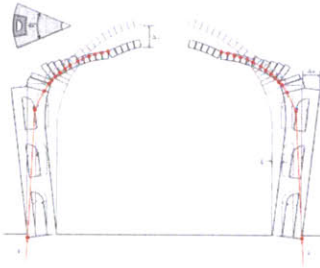
<sup>98</sup> Jacobson, *Hadrianic Architecture and Geometry*, pp.69-85.

<sup>99</sup> Rivoira, *Roman Architecture and its Principles of Construction under the Empire*, 118-159.

<sup>100</sup> McEwen, *Vitruvius*, 212-229.

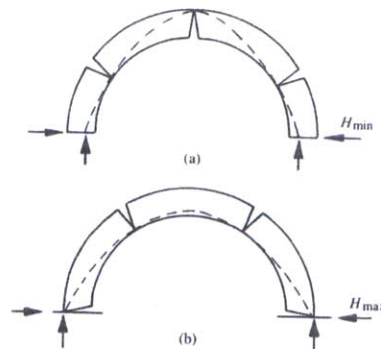
<sup>101</sup> Rivoira, *Roman Architecture and its Principles of Construction under the Empire*, 118-159.

dome.<sup>102</sup> Zessin expanded on the idea by characterizing the failure mode of the Pantheon's dome on spreading supports.<sup>103</sup>



**Figure 5.4 - Zessin's model of dome collapse on spreading supports found this critical angle of lean for the Pantheon.**

Failure begins at points where the thrust line of an arch meets the boundary formed by the inner or outer edge of a dome. This can happen at a minimal thrust state or a maximal thrust state as shown in Figure 5.5. These points, referred to as hinges, will not immediately cause failure. Instead, the local stresses will cause cracks and the neighboring material will settle into a new stable configuration. However, once the local stresses exceed the hinged equilibrium state, the dome will collapse.<sup>104</sup> Nevertheless, scholars agree that the Pantheon is not in danger of collapsing anytime soon.



**Figure 5.5 - The top figure shows the hinge points in an arch in a minimum thrust state. The bottom figure shows the hinge points for an arch in a maximum thrust state.**

The *dome\_thrust\_line* model was able to find a stable equilibrium state for a dome of similar characteristics as the Pantheon's dome. The dome of the Pantheon features graded density along the height of the dome and the *dome\_thrust\_line* model accepts only a single density. However, if a dome with uniform density (equal to the highest density in the Pantheon) can have a stable thrust line, then it is a reasonable conclusion that a dome with decreased density at the crown will also have at least one stable thrust line. That the Pantheon's dome is made of lighter material towards the crown is nevertheless an important innovation in architecture. While it may

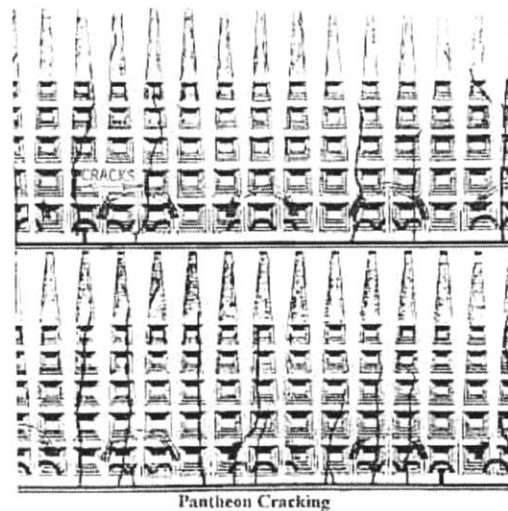
<sup>102</sup> Lancaster, *Concrete Vaulted Construction in Imperial Rome*, 158-161.

<sup>103</sup> Zessin, *Collapse Analysis of Concrete and Masonry Domes*, 41-50.

<sup>104</sup> Heyman, *The Stone Skeleton*, 35.

not have been a necessary feature, it makes the dome more durable in the long term because the overall stresses placed on the supports are significantly lower.

The cracks in the Pantheon, resulting from strain of natural settlement, have been worsened by vibrations as Rome developed around it. Terenzio, an administrator in charge of Roman monuments, documented the cracks in the Pantheon's rotunda.<sup>105</sup> Moore's study of these cracks in the supporting piers of the Pantheon uses material properties to confirm the measure stresses in the cracked piers. His main project involves replicating Roman concrete so that material properties can be measured. Despite having made many different concrete recipes, he has yet to find the recipe that yields concrete of the same properties as the concrete used in Roman buildings. This is especially difficult in the case of the Pantheon, where the density of the dome is estimated to change by as much as 30%. Using the sample properties from a relatively similar concrete recipe, Moore was able to predict that the concrete in the piers would crack in the same locations as the existing cracks.<sup>106</sup>



After Terenzio<sup>22</sup>

Figure 5.6 - The cracks in the rotunda of the Pantheon.

The model *dome\_thrust\_line* also calculates a thrust force using material properties. In the trial run above, the material properties that Moore used in his study for concrete of density 1600kg/m<sup>3</sup> were used and yielded the same estimated thrust force calculated by Moore. The same material properties were again used in *wall\_load* to find the maximum allowable load of the walls and piers of the Pantheon. Finally, the load due to the thrust force and weight of the dome was compared to the results *wall\_load* to confirm that the dome load was within the strength capacity of the walls and piers.

<sup>105</sup> Terenzio, *La restauration du Pantheon de Rome*, 103-105.

<sup>106</sup> "The Pantheon," <http://www.romanconcrete.com/docs/chapt01/chapt01.htm>.



The results are summarized in the table below.

**Table 5.7 - Results of the Pantheon dome analysis, including a previously calculated load for comparison.**

<b>Stable Thrust Line Found?</b>	<b>Estimated Load [N]</b>	<b>Estimated Load [kg]</b>	<b>Load (Moore) [kg]</b>	<b>Maximum Compressive Load [N]</b>	<b>Maximum Tilt Load [N]</b>
<b>True</b>	559.26E3	4.474E6	4.535E6	2.377E9	1.517E6

## 5.2 The Teatro Marittimo

The Teatro Marittimo is a building located in Hadrian's Villa at Tivoli. The construction of the villa was a personal project of Hadrian that took an estimated two decades to build during the 2nd century AD. It was in this villa that Hadrian was able to experiment with his own architectural designs that had been previously mocked. A map of the villa can be found in Figure 5.8.<sup>107</sup>

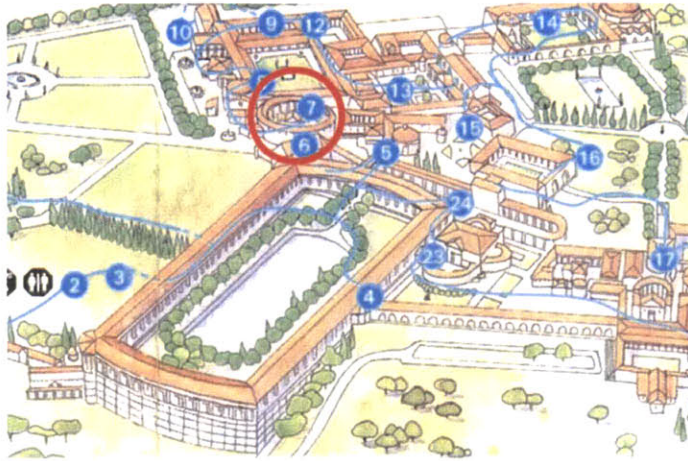


Figure 5.8 - The Teatro Marittimo, marked by the red circle, is shown among the main cluster of buildings in Hadrian's Villa.

At the heart of the main building cluster, the Maritime Theater was Hadrian's personal retreat within a retreat, not a theater as implied by the name.<sup>108</sup> Its form is similar to the stereotypical medieval castle in that the building is isolated by a moat-like canal. The circular building stood at the center of the canal like an island. The building contained all the rooms one would expect to find in a typical Roman villa, including tablinum, bedrooms, a dining room, and a library. Most of the remains are of walls and columns. Some pieces of the entablatures also remain and have been reconstructed over their respective columns.

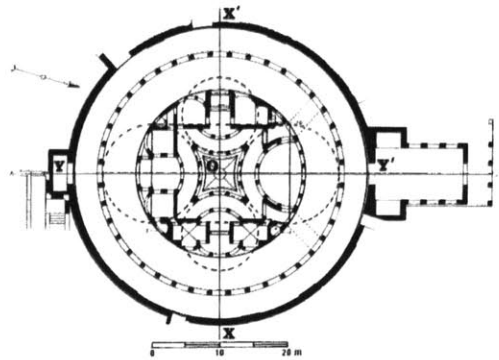
<sup>107</sup> Rivoira, *Roman Architecture and its Principles of Construction under the Empire*, 118-159.

<sup>108</sup> Rivoira, *Roman Architecture and its Principles of Construction under the Empire*, 118-159.



**Figure 5.9 - A view of the reconstructed columns around the outer canal ring of the Teatro Marittimo.**

Like the Pantheon, the predominant geometry of the floor plan is based on Euclidean geometry.<sup>109</sup> Euclidean geometry used a single radius as the basis for the placement of circles along a single coordinate axis. The dimensions of all the circles, and their displacement from the central circle, were proportionally related to the base radius. Even the columns were arranged in the circular patterns.

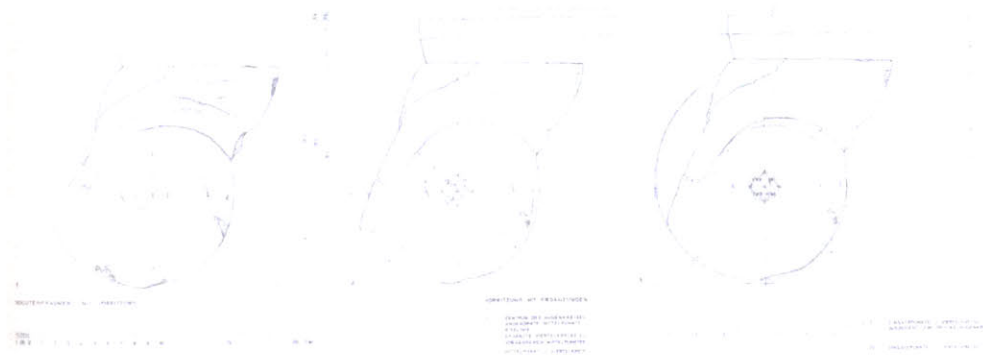


**Figure 5.10 - A ground plan of the Teatro Marittimo showing proposed Euclidean geometry basis.**

Nevertheless, the columns show traces of Vitruvian moduli. Ueblacker's extremely detailed study of the Teatro Marittimo includes a close look at the capitals of the columns. In *De architectura*, Vitruvius uses the proportions of man and Greek ratio mathematics to derive proportion guidelines for columns. One of these is a proportion recognized as the golden ratio. Vitruvius proposes that volutes formed by the logarithmic spiral based on the golden ratio (known as the golden spiral) are perfect and coherent like the circle and square because they are ultimately connected to the fundamental numbers. Ueblacker confirmed that the volutes in the columns of the Teatro Marittimo are based on the golden spiral.<sup>110</sup>

<sup>109</sup> Jacobson, *Hadrianic Architecture and Geometry*, pp.69-85.

<sup>110</sup> Ueblacker, *Das Teatro Marittimo in der Villa Hadriana*, 39-42.



**Figure 5.11 – Here, Ueblacker shows three different possible volute spirals, all of which begin with the golden spiral.**

There are four distinct columns in the Teatro Marittimo. These columns are found in the outer canal ring, the tablinum, the exedra, and the peristyle. They are all Ionic in base and capital, and are fluted. Ueblacker has also confirmed that the number of flutes corresponds to the number of flutes that Vitruvius predicts for columns of their respective diameters.<sup>111</sup> The height of the reconstructed columns corresponds to the modular height calculated by their respective diameters. In addition, the pieces of the entablatures that remain also comply with the diameter-based proportions set by Vitruvius. Based on this finding, all the missing entablatures were also assumed to follow Vitruvian proportions in the analytical models of the columns. In addition, the intercolumnar spacing rules that Vitruvius proposed in *De architectura* are meant to be applied in squared buildings. Nonetheless, the spacing of the columns, measured along the circumference of the base Euclidean circle and reduced to moduli based on the column diameters, is fairly close to the spacing proposed by Vitruvius.

<sup>111</sup> Ueblacker, *Das Teatro Marittimo in der Villa Hadriana*, 39-42.





Figure 5.12 - Left to right, the columns in the outer canal ring, the tablinum, the peristyle, and the exedra are marked.

The *estimate\_columnar\_load* model takes the diameter and height of a column and uses the Vitruvian proportions given in Table # to estimate the volume of rock that would rest on a single column. The columns were cut from stone at the quarry known as the Cave di Cusa. According to an Italian geological survey, the predominant type of rock in the area of Cusa is a type of travertine limestone. Therefore, *estimate\_columnar\_load* uses the material properties of limestone to calculate the weight of the entablature that would rest on a single column.

The model *column\_buckling* then calculates the critical loads and safety factor of the column. Using the method that finds the self-crushing height of a limestone column, the buckling load is calculated for the column based on the compressive yield stress. Vitruvius proposed that columns and walls should be built such that they can carry three times the load as a safety measure. Modern construction projects feature safety factors between 1 and 2 in design though in practice the resulting safety factor can be closer to 0.75. Using the Vitruvian safety factor of three, the maximum allowable load was also calculated. A comparison of the estimated load, buckling load, and allowable load proves that all of the columns are stable. Interestingly, the actual safety factors are estimated to be less than the factor of three Vitruvius wanted, but the columns are still structurally sound.

The results of the column analysis are summarized in the tables below.

Table 3.13 - The calculated self-crushing heights and Vitruvian rule-check results for each column.

Column	Conforms to Vitruvian rules?	$H_{crush}$ [km]
Canal	True	25.570
Tablinum	True	25.570
Peristyle	True	25.570
Exedra	True	25.570

Table 5.14 - The estimated, buckling, and allowable (by Vitruvius' rules) load and stress for each column, as well as the estimated safety factor.

Column	$P$ [MN]	$P_{cr}$ [MN]	$P_{allow}$ [MN]	$\sigma$ [MN/m <sup>2</sup> ]	$\sigma_{cr}$ [MN/m <sup>2</sup> ]	$\sigma_{allow}$ [MN/m <sup>2</sup> ]	$FS$
Canal	104.530	138.210	34.842	380.77	503.47	126.92	0.756
Tablinum	29.243	74.413	24.804	189.39	481.91	160.64	2.545
Peristyle	33.807	47.625	15.875	218.94	308.43	102.81	1.409
Exedra	8.665	9.407	3.136	126.26	137.08	45.693	1.086

### 5.3 The Sala dei Filosofi

The Sala dei Filosofi is located near the Teatro Marittimo in Hadrian's Villa at Tivoli. While the Teatro Marittimo had its own, smaller, library, Hadrian built a much larger library near his island retreat.<sup>112</sup> This library, known as the Hall of Philosophers, has a floor plan that shows elements of the traditional squared design as well as Euclidean circular design.<sup>113</sup> The eastern entrance connects to the Teatro Marittimo while the western entrance leads to the colonnaded *Poikile*.



**Figure 5.15 - The Sala dei Filosofi (in color) is located behind the Teatro Marittimo.**

While the Teatro Marittimo was chosen for its columns, the Sala dei Filosofi was chosen for the dome and walls. Very few scholars have devoted their attention to the library and none have conducted quantitative studies as of yet. Ricotti has studied the bricks of the Sala dei Filosofi in enough detail to determine that they were manufactured and laid according to the methods described in *De architectura*.<sup>114</sup> The analysis of this structure was conducted using the raw data collected in a project led by the architects Mangurian and Rayburn.



**Figure 5.16 - The semi-dome at the back of the room. The roof may have continued into a vault over the entire chamber.**

<sup>112</sup> Rivoira, *Roman Architecture and its Principles of Construction under the Empire*, 118-159.

<sup>113</sup> Rivoira, *Roman Architecture and its Principles of Construction under the Empire*, 118-159.

<sup>114</sup> Ricotti, *Villa Adriana*, 13.

The process was the same as the process used to analyze the Pantheon. The *wall\_load* model was used to calculate the maximum load of the walls that support the vault. Then the dimensions of the vault were used in the *dome\_thrust\_line* model. The program found a stable equilibrium state and calculated the corresponding thrust force. The materials used in the bricks that were used in the construction of the Sala dei Filosofi's walls and vault were identified by Ricotti.<sup>115</sup> The bricks contained limestone and sandstone as aggregates. Sandstone is the weaker of the two stones; therefore the materials properties of the bricks were estimated to be similar to sandstone properties. A comparison of the results from both programs shows that the load from the vault on the walls is well within their capacity.

The results are summarized in the table below.

Table 5.17 - The results of the Sala dei Filosofi wall and dome analysis.

Stable Thrust Line Found?	Estimated Load [kN]	Maximum Compressive Load [kN]	Maximum Tilt Load [kN]
True	4.730	25.570	0.671

## 5.4 Conclusions

In conclusion, the three buildings constructed under Hadrian that were analyzed for this investigation are the Pantheon, the Teatro Marittimo, and the Sala dei Filosofi. The Pantheon was used as a test case to confirm that the models functioned and provided reasonable results.

The Teatro Marittimo provided opportunities for a qualitative and quantitative comparison to Vitruvius. As Ueblacker has concluded, the Teatro Marittimo incorporates many Vitruvian principles and proportions despite the Euclidean base design. The principle element of analysis for the Teatro Marittimo is the column. Four column designs were identified and all of them follow Vitruvian modular proportion based on the column diameters. In addition, the structural analysis shows that the columns and, by association, Vitruvius' proportions are structurally sound. The Sala dei Filosofi's vault rests on a wall that was built according to the procedures described by Vitruvius in *De architectura*. In addition, the loading of the wall is structurally stable.

While Vitruvius did not invent the procedures he outlines, the goal of the analysis was to confirm that Vitruvius was accurately reflecting Roman construction practices. Thus, the presence of Vitruvian practices in buildings built a century later shows that despite the innovations, Roman architecture continued to rely on the basic principles described in *De architectura*. In addition, the structural analysis of the columns and walls shows that these practices are structurally stable.

<sup>115</sup> Ricotti, *Villa Adriana*, 57-110.



## CONCLUSION

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Vitruvius had three unifying concepts, fundamental to all Roman construction, that tied together all of his proposed standards of architecture. These concepts represented the most simplified ideals of practicality and beauty in Roman architecture. These three concepts are: *firmitas*, *utilitas*, and *venustas*. Under these three umbrella ideas, Vitruvius further listed seven characteristics of integrity, strength, function, and beauty.<sup>116</sup> These concepts were distinct but not separable, nor are they merely inventions of Vitruvius' mind. Vitruvius, in accordance with his most basic goal of presenting a collection of knowledge, condensed a general sense of architectural ideals into discrete criteria.

The principle of *firmitas* dictates that a building must be strong and stable. To achieve *firmitas*, a building must correctly apply the principles of *dispositio*, *distributio*, *ichnographia*, and *ordinatio*. The principle of *venustas* joins the practical nature of construction with the aesthetic and philosophical values of Roman architecture. The proper use of *decor* and *symmetria* allowed architects to alter the underlying geometry of a building to make it more aesthetically pleasing and achieve *venustas*. Finally, the principle of *utilitas* states that the design of the building should clearly reflect its purpose. Fundamental to *utilitas*, is the concept of *eurythmia*, which describes the cohesive nature of the building's individual elements.<sup>117</sup>

### 6.1 Utilitas

Roman architecture rigidly adhered to *utilitas*. Looking at the floor plan of a building often provides enough clues as to its purpose. However, this kind of utility appears to be more of a universal architectural design constraint than a new concept proposed by Vitruvius. In fact, Vitruvius himself credits the concept to older Greek architects. In this case, Vitruvius merely presents a clear and concise definition of *utilitas*. He also states that *utilitas* should always be evident in a construction project. This is the case for most buildings, including the three studied in this thesis. The Pantheon is most likely a temple, the Teatro Marittimo is most likely a home, and the Sala dei Filosofi is most likely a library. *De architectura* is relevant to all Roman architecture with regards to *utilitas*.

### 6.2 Venustas

*Venustas* is trickier to quantify because it involves the very subjective nature of beauty. Vitruvius' precepts all contribute to the final development of *venustas*, but Vitruvius carefully qualifies the rules by saying that adjustments to dimensions are possible, and expected, to improve the final aesthetic quality of a building. The dimensions of the columns and walls in the test cases fall within the modular rules set by Vitruvius, but are not exact. This implies that architects followed the dimensional modules but also understood Vitruvius' qualification. In a chicken-and-egg type of paradox, no one knows if Vitruvius is the source or the channel for these rules. There is

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<sup>116</sup> Pollio, *Vitruvius on Architecture*, ed. Smith, 13-25.

<sup>117</sup> McEwen, *Vitruvius*, 72.

evidence that the rules were classified by styles of Greek architecture, but there is enough variation that they cannot be the source of Vitruvius' modules. Regardless of which came first, *De architectura* presents rules that were followed even a century later.

### 6.3 Firmitas

Several models were used to explore the application of the more quantitative, though not necessarily explained, guidelines which Vitruvius proposed for the design of buildings. Vitruvius limited his quantitative precepts to the formation of floor plans and construction of supporting structures. Malacrino, as well as others, have explored the materials and techniques of the construction of supporting structures, particularly the foundations of buildings. Malacrino is able to trace Hellenistic roots and influences on building foundations in Rome, as well as show how Vitruvius was applied in their construction.<sup>118</sup> The foundation materials were always chosen and laid in a manner considerate of the local climate and soil, as prescribed by Vitruvius.<sup>119</sup> Looking above the foundations, the main supporting structural elements of a building are the columns and the walls.

#### 6.3.1 Columns

Vitruvius summarized different styles of colonnades where the height and inter-columnar spacing depended on the type of column (Ionic, Doric, or Corinthian) and the diameter of the columns. Although he does not explain his reasoning quantitatively, he does explain that these guidelines are characteristic of certain styles and that sticking to one style helps make a building a cohesive structure. Vitruvius also proposed guidelines regarding the heights of the individual components of an entablature which depended on the height of the column (and by the previous relationship also depended on the diameter and type of column). Vitruvius also suggested that the height of a column should be limited such that the height would be able to support up to three times the load without crumbling in on itself.<sup>120</sup> In essence, Vitruvius was warning against building a column higher than its self-crushing height or loading a column with the equivalent force of that height. In this study, it was checked that columns, and their entablatures, were constructed following these guidelines, and whether the same rules had any engineering merit. Using the buckling models described before, four types of columns were analyzed in the Teatro Marittimo. These columns are located in the canal ring, the peristyle, the tablinum, and the exedra.

It was found that only the columns in the tablinum adhered to the proposed rules of diameter, height, and base. In addition, only the entablatures of the canal ring columns and exedra columns have been reconstructed, but those reconstructions appear to conform to Vitruvian guidelines. Having noticed this, the assumption was made that the entablatures of the columns in the peristyle and tablinum also conformed to Vitruvius' rules. Under this assumption, it was found that the loads placed on the columns by the entablatures were well within the capacity of the columns' axial strength. Having determined the maximum axial load of a column,

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<sup>118</sup> Malacrino, *Constructing the Ancient World*, translated by Hyams, 111-113.

<sup>119</sup> Malacrino, *Constructing the Ancient World*, translated by Hyams, 66-67.

<sup>120</sup> Vitruvius, *De architectura*, 3-4

the model then calculated the self-crushing height using the procedure described by Heyman.<sup>121</sup> As expected, the self-crushing height of limestone columns is greater than 2 kilometers.

It is important to note that the buckling model relies heavily on material properties, which are largely unknown. The columns in the Teatro Marittimo are thought to have originated from a quarry known as the Cave di Cusa. Several unfinished columns have been found in the quarry that matches the style and time period of the construction in Hadrian's villa. No source explicitly describes the qualities of the stone from this quarry, and tests have not been allowed on the columns themselves for fear of destroying them. Therefore, geological survey data from the Italian government was used to determine that the predominant type of rock in the area of Cusa is limestone. The buckling model therefore uses estimated material properties of limestone when calculating the entablature load and column maximum load.

### 6.3.2 Walls

A similar analysis was used to characterize load-bearing walls. This wall model takes the material properties of a wall, as well as the dimensions, and calculates the maximum load the wall can bear on its vertical axis, the force required to tilt the wall, and the wind speed at which the wall would collapse. This model was used to analyze the walls of the Pantheon and the Sala dei Filosofi. Ultimately, the results from this analysis were compared to the results from the thrust line analysis to determine that the supporting walls were capable of balancing the loads from vaulted structures.

### 6.3.3 Vaulted Structures

Vitruvius never mentions vaulted construction in *De Architectura*, except to set forth a few guidelines regarding the proportions and placements of vaults in basilicas. However, vaulted construction, particularly concrete vaulted construction, is yet another one of ancient Rome's legacies. Vaulted construction relies heavily on the supporting structures of the vaults; the columns and walls must be able to bear not only the axial loads, but also the thrust forces resulting from the non-linear structure they support.<sup>122</sup> In addition, vaulted structures became more prominent just as floor plans began to move away from the square and circle designs that became traditional following Vitruvius' work. During Hadrian's lifetime, and up until the time of Justinian, floor plans began to follow Euclidean geometry by using the placement of circles to form concave and convex spaces.<sup>123</sup> This new direction of architecture did not, however, render Vitruvius outdated. His methodology of defining strength and safety continued to be used and remained relevant when vaulted structures were built on columns and walls.

The analysis of vaulted construction is tricky business. Recent models to analyze vaulted structure stability rely mostly on thrust line analysis and the division of vaults and domes into lunes. Using this methodology, a simplified thrust line analysis model was developed to find a

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<sup>121</sup> Heyman, *The Stone Skeleton*, 12.

<sup>122</sup> Lancaster, *Concrete Vaulted Construction in Imperial Rome*, 110.

<sup>123</sup> Jacobson, *Hadrianic Architecture and Geometry*, pp.69-70.

single stable state. According to the master safe theorem of masonry, as long as at least a single equilibrium state can be found, the structure can be deemed stable, even if the thrust line calculated is not the actual thrust line in the structure.<sup>124</sup> In addition, the vaulted model also calculates the thrust forces and weight of a dome or vault.

The Pantheon's dome and thick, granite walls were used as a control to determine the accuracy of the models, while the vault and walls of the Sala dei Filosofi were previously untested in this manner. A comparison of the results of the wall tests and the vault tests reveals how well the supporting structures, in both cases, walls, can support the loads from the vaulted structures. The tests confirmed that Pantheon's dome is stable and that the forces from the dome on the supporting walls are within the axial and tilting capacities of the walls. The weight thrust forces were compared against previous results from Moore's work and are within 2% of Moore's calculated forces. The discrepancy is probably due to the fact that the model is not programmed to account for the non-uniform density of the Pantheon's dome while Moore was able to make his more advanced model gradate the concrete density by height. In addition, both estimations rely on estimated material properties of ancient Roman concrete. Since current methods to determine material strengths would destroy samples, no one has been able to test samples from well-known Roman buildings. The material properties used in this model were estimated by Cowan from samples taken from Roman concrete ruins in Libya.<sup>125</sup>

The material properties used for the models of the Teatro Marittimo and Sala dei Filosofi are based on the probable origin of the stones (Cave di Cusa) and identifications of brick materials made by Ricotti. The columns in the Teatro Marittimo show evidence of following Vitruvian modular dimensions. Though not all the entablatures remain, the surviving pieces show great similarity to Vitruvian guidelines. Therefore, it seems reasonable to conclude that all the column entablatures adhered to the same constraints. The columns were found to be stable and more than strong enough to carry their respective loads. The vault in the Sala dei Filosofi was found to be stable. The walls of the Sala dei Filosofi are capable of bearing the weight of the vault and can remain stable despite the thrust forces. The safety factor for the wall is also greater than three.

Based on the results of the structural models, it appears that Vitruvius' rules pertaining to *firmitas* were generally adhered to in both Augustan and Hadrianic construction. These rules are overly conservative and in practice, yielded less conservative structures. Despite this, the rules provide for robust structural engineering practices.

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<sup>124</sup> Heyman, *The Stone Skeleton*, 22.

<sup>125</sup> Cowan, *The Master Builders*, p. 73.

## APPENDIX A: MATLAB FILES

---

### A.1 estimate\_columnar\_load

```
function [ P ] = estimate_columnar_load( D, H, A, F, C, I )
%ESTIMATE_COLUMNAR_LOAD Returns the load on a column.
% Function uses the column diameter and height to determine the
% characteristics of the entablature as set by Vitruvius. It then
% calculates the load of the entablature on a single column.

%parameters
rf = 0.2956;
ro_lime = 2700; %kg/m3
I_m = I*D*rf;
D_m = D*rf;
H_m = H*rf;

if (H>=12) & (H<=15)
    %architrave
    if (A == 1)
        H_arch = 50;
    elseif (A ==0)
        H_arch = 0;
    end
    %frieze
    if (F == 1)
        H_f = 37.5;
    elseif (F == 2)
        H_f = 62.5;
    else
        H_f = 0;
    end
    %cornice
    if (C == 1)
        H_c = 14.28;
    elseif (C ==0)
        H_c = 0;
    end
end

if (H>15) & (H<=20)
    %architrave
    if (A == 1)
        H_arch = 69.23;
    elseif (A ==0)
        H_arch = 0;
    end
    %frieze
    if (F == 1)
        H_f = 51.92;
    elseif (F == 2)
        H_f = 86.53;
    else
```



```

        H_f = 0;
    end
    %cornice
    if (C == 1)
        H_c = 19.78;
    elseif (C == 0)
        H_c = 0;
    end
end

if (H>20) & (H<=25)
    %architrave
    if (A == 1)
        H_arch = 72;
    elseif (A == 0)
        H_arch = 0;
    end
    %frieze
    if (F == 1)
        H_f = 54;
    elseif (F == 2)
        H_f = 90;
    else
        H_f = 0;
    end
    %cornice
    if (C == 1)
        H_c = 20.57;
    elseif (C == 0)
        H_c = 0;
    end
end

if (H>25) & (H<=30)
    %architrave
    if (A == 1)
        H_arch = 75;
    elseif (A == 0)
        H_arch = 0;
    end
    %frieze
    if (F == 1)
        H_f = 56.25;
    elseif (F == 2)
        H_f = 93.75;
    else
        H_f = 0;
    end
    %cornice
    if (C == 1)
        H_c = 21.43;
    elseif (C == 0)
        H_c = 0;
    end
end

```

```

end

%total entablature height and volume
H_tot = (H_arch + H_f + H_c)*D;
H_tot_m = H_tot*rf;
v = H_tot_m*I_m^2;

%total load on column
P_N = v*ro_lime*9.8; %N
P_kg = v*ro_lime; %kg
P = [P_N, P_kg];

end

```

## A.2 column\_buckling

```
function [ H_crush, P_final, sigma, F_safe, valid] = column_buckling( D, H,  
P )  
%COLUMN_BUCKLING Returns loading & safety information and predicts  
%   Function uses column parameters, in roman feet, to determine the  
%   self-crushing height of a column, the maximum/allowable load, the  
%   maximum/allowable axial compressive stresses, and the safety factors.  
  
%D = lower diameter of column, rf  
%H = height of column, rf  
%P = load, N  
%E = modulus of elasticity  
%K = effective length factor  
%A = area, m2  
%lrf = 0.2956m  
  
%parameters  
rf = 0.2956;  
D_m = D*rf;  
H_m = H*rf;  
r = D_m/2;  
L_r = H_m/r;  
v = pi*H_m*r^2;  
a = pi*r^2;  
I = (pi*r^4)/4;  
  
%material properties  
ro_lime = 2700; %kg/m3  
sigma_lime_compress = 200E6; %N/m2  
E_lime = 50E9; %N/m2  
  
%calculate self-crushing height  
w = ro_lime*9.8; %N/m3  
H_crush_rf = sigma_lime_compress/w; %rf  
H_crush_m = sigma_lime_compress/w/rf; %m  
H_crush = [H_crush_m, H_crush_rf];  
%check model valid  
sigma_slender = (v*ro_lime*9.8)/a;  
if (sigma_slender <= sigma_lime_compress)  
    disp('Euler Model Valid')  
    valid = 'Valid';  
else  
    valid = 'Invalid';  
end  
  
%find critical load  
P_cr_N = ((pi^2)*E_lime*I)/(H_m^2); %N  
P_cr_kg = P_cr_N/9.8;  
s_crit = P_cr_N/a;  
  
%find current load
```

```

FS_current = P_cr_N/P(1);
FS_conservative = 3;
s_allow = s_crit/FS_conservative;
P_allow_N = s_allow*a;
P_allow_kg = P_allow_N/9.8;
s_estimated = P(1)/a;

%store values
P_final = [P(1), P(2), P_cr_N, P_cr_kg, P_allow_N, P_allow_kg];
F_safe = [FS_current, FS_conservative];
sigma = [s_estimated, s_crit, s_allow];

end

```

### A.3 dome\_thrust\_line

```
function [ thrust, equilibrium ] =  
dome_thrust_line( alpha, theta, phil, n, R, t, roro, round)  
%DOME_THRUST_LINE Finds a single possible thrust line.  
% Function will find a single possible thrust line and return the  
% computed thrust as well as a true value if the thrust line is in  
% equilibrium.  
  
close all;  
  
% parameters  
seg = round/theta;  
alpha = (alpha*pi)/180;  
theta = (theta*pi)/180;  
t_R = t/R;  
ri = R - (t/2);  
ro = R + (t/2);  
  
% determine thrust  
% initialize min thrust  
thrust = 10^10;  
  
% discretization  
m_max = R/2;  
k_max = m_max;  
d_phi = 2*alpha/n;  
  
[x_cg_tot, v_tot] = lune(ri, ro, theta, phil, alpha);  
  
% search for a valid min thrust state  
for k = 1:k_max+1  
    for m = 1:m_max+1  
  
        % define thrust line position at crown and springing  
        rise = ro-t*(k-1)/k_max;  
        run = ro-t*(m-1)/m_max;  
  
        % calc thrust, T, for given thrust line constraints  
        T = v_tot*(run*sin(alpha)-x_cg_tot)/(rise-run*cos(alpha));  
  
        % define x,y coordinates for thrust line  
        max_count = ceil(n/2)+1;  
        xy=zeros(max_count,2);  
        r=zeros(max_count,1);  
        xy(1,:) = [0,rise];  
        r(1,:) = rise;  
  
        %define phi2  
        if mod(n,2)  
            phi2_2 = d_phi/2;  
        else  
            phi2_2 = d_phi;  
        end  
    end  
end
```



```

% determine centroid and volume coordinates
[x_cg, volume] = lune(ri, ro, theta, phi1, phi2_2);
r(2,:) = (T*rise+volume*x_cg)/(volume*sin(phi2_2)+T*cos(phi2_2));
xy(2,:) = r(2)*[sin(phi2_2),cos(phi2_2)];

%voussoirs
for j = 3:max_count

    phi2 = phi2_2 + d_phi*(j-2);

    [x_cg, volume] = lune(ri, ro, theta, phi1, phi2);
    r(j,:) = (T*rise+volume*x_cg)/(volume*sin(phi2)+T*cos(phi2));
    xy(j,:) = r(j)*[sin(phi2),cos(phi2)];
end

% check validity of thrust state
valid = range(ri, ro, r);
%store values
if (valid == 1) && (T < thrust)
    equilibrium = 'true';
    thrust1 = T*roro; %thrust
    xy_final_min = xy; %final xy coordinate
    r_final_min = r; %fine r coordinate
end
end

end

thrust2 = thrust1*seg;
thrust = [thrust1, thrust2];

end

```

## A.4 wall\_load

```
function [ F_tilt_cog, F_compress, v_m ] = wall_load( L, W, H, ro, sigma )
%UNTITLED Summary of this function goes here
% Detailed explainaion goes here

rf = 0.2956;
%lpsf = 47.88 N/m2
psftonmm = 47.88;
cd = 2.0;

L_m = rf*L;
W_m = rf*W;
H_m = rf*H;
a = L_m*W_m;
v = L_m*W_m*H_m;
m = v*ro;
I = L_m*(W_m^3)/12;

cog_x = W_m/2;
cog_y = H_m/2;
F_tilt = m*9.8;
F_tilt_cog_N = m*9.8*cog_x;
F_tilt_cog_kg = m*cog_x;
F_tilt_cog = [F_tilt_cog_N, F_tilt_cog_kg];

F_compress_N = sigma*a;
F_compress_kg = sigma*a/9.8;
F_compress = [F_compress_N, F_compress_kg];

Kz = (cog_y/33)^(2/7);
Gh = 0.65 + (0.60/(H_m/33))^(1/7);
% F = A*PSF*cd*Kz*Gh*psftonmm;
% PSF = 0.00256*v^2
PSF = F_tilt/(a*cd*Kz*Gh*psftonmm);
v = sqrt(PSF/0.00256);
v_m = v*0.44704;

end
```

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